CHAPTER ONE

INTRODUCTION

1.0 Background

The thermodynamics analysis involving channel fluid flow has attracted a lot of researches all over the world. This is due to the fact that the knowledge of thermodynamics has enabled us to deal with the analysis of machines, which are used to convert chemical or nuclear energy into useful work [1, 2]. Thermodynamics of fluid flow are important tools that help to analyse new flow systems [3]. Magneto hydrodynamics (MHD) is one of the important aspects of fluid dynamics, which has attracted a lot of researchers. MHD is the field of study that explains the physics of electrically conducting fluid that results from the interaction of magnetic field with the fluid flow.

In this project, we have considered the MHD laminar flowing through a porous medium with a velocity u in a horizontal channel of finite depth H, in the presence of a uniform magnetic field B_0 [2]. The statement of the problem is stated in the next subsection.

1.1 Statement of the Problem

This project is to study the effect of the magnetic field of magnitude B_0 of the fluid that is flowing within a channel, whose width is H. The fluid is allowed to flow through a porous medium with certain boundary conditions. The fluid motion is initialised by the pressure gradient along the flow direction. The pressure gradient can also be referred to as the pump in flow system. With this flow system, we study the implication of the magnetic field on the fluid flow.

1.2 Objective of the Study

The objectives of this project are to:

1. Recast the equations of the fluid motion into their dimensionless forms.
2. Solve the recast equations with the Adomian Decomposition Method (ADM).
3. Implement the ADM recursive scheme in the symbolic package MATHEMATICA.

1.3 Significance of the Study

The results to be obtained from this project are expected to help advance many industrial and thermal engineering processes, where the science of fluid flow are applied.

1.4 Project Outline

Chapter 2 contains the literature, relevant to the problem, where some properties of fluid and the classifications of fluid flow are reviewed. The Adomian decomposition method, which we applied to obtain the solution to the governing equations, in this project is explained. Chapter 3 contains the methodology used in the project. We present and discuss the results in chapter 4, while conclusions and recommendation are discussed in chapter 5.

CHAPTER TWO

LITERATURE REVIEW

2.0 Introduction

In this project, we employ a fundamental principle of physics, which is used to derive the governing differential equations in fluid mechanics or fluid dynamics. But before that, we look into some properties of fluid and its classifications; this will help us understand and get some basic things about fluid. This principles are the conservation principles, which include the principle of conservation of mass (continuity equation), the principle of conservation of linear momentum (Newton's second law), and the principle of conservation of energy (First law of thermodynamics). In order to apply these principles in fluid mechanics or fluid dynamics, it is usual to imagine a small portion of fluid; this does not change as it moves from a point to another within the volume of the fluid. The small portion of the fluid is called the fluid parcel which is known as the basis for the continuum hypothesis. The application of the principles, mentioned above, and the continuum hypothesis would be presented in the following subsections.

2.1 Fluid

Fluid mechanics is an important area in physics. Life as we all know it would not exist without fluids and without the behaviour that fluids exhibit. Fluid plays a vital role in our everyday life. Primarily we classify the states of matter into three categories namely solids, liquids and gasses. However gasses and liquid seems to be different because they have certain properties in common which differentiates them from solids. A fluid is generally a substance that flows. That is, fluids that cannot withstand a tangential force to its surface and hence that takes the shape of a can or a container [8]. Solid can resist the tangential force deforming its shape whereas fluid deforms continuously under the influence of shear stress, no matter how its shape.

2.1.1 Properties of fluids

Pressure (p): It is the normal force exerted by a fluid per unit area at a particular point. The SI unit system and the dimension of pressure is expressed as, N/m^2 and $ML^{-1}T^{-2}$, respectively.

Density (ρ): The density of a substance is said to be the quantity of matter contained in unit volume of the substance [5]. It is expressed in three different ways, which is known to be the mass density, specific weight (ρg) and the relative density or specific gravity.

The mass density is expressed as;

$$\left[\rho = \frac{\text{mass}}{\text{volume}}\right]$$

While the relative gravity/specific gravity;

$$\left[SG = \frac{\rho}{\rho_{water}}\right]$$

For the mass density; Dimension: ML^{-3} unit: kg/m^3

For the specific weight; Dimension: $ML^{-2}T^{-2}$ unit: N/m^3

The standard value for density of water and air are given as $1000 kg/m^3$ and $1.2\ kg/m^3$, respectively.

Viscosity: Is the internal friction in fluid; viscous forces oppose the motion of one portion of a fluid relative to another [9]. Viscosity is also the resistance of any fluid (liquid or gas) to a change in shape, or flow of neighbouring portions relative to one another.

$$f = \mu A \frac{u}{y}$$

Where f is known as the force, μ is the viscosity of the fluid, A is area of each plates and $\frac{u}{y}$ is the rate of shear deformation, we also explain the two different types of viscosity, which are dynamics viscosity and kinematic viscosity. First, we talk about the dynamic viscosity, which is the tangential force per unit area needed to move the fluid in one horizontal plane with respect to other plane. The dynamic viscosity formula can be expressed as;

$$\mu = \frac{T}{\gamma}$$

Dynamic viscosity = shear stress / shear rate

The μ is known as the dynamic viscosity, the *T* is known as the shear stress and the γ is the shear rate, in addition, the kinetic viscosity is the ratio between the dynamic viscosity and the mass density. It is also the measurement of the internal resistance when the fluid is in motion [10].

$$v = \frac{\mu}{\rho}$$

The v is known as the kinematic viscosity, and the ρ is the mass density, the μ is the dynamic viscosity. The kinematic viscosity SI unit is in s/m^2 (square meter per second).

Temperature and Ambient temperature (T): Temperature is the degree of hotness and coldness of a system. Most time temperature is expressed in centigrade (℃) where the freezing point and boiling point of a fluid mostly water is taken from (0℃) and (100℃). The SI unit of temperature is expressed in term of Kelvin. We also talk about the ambient temperature. This is the average air temperature surrounding something (such as a person) whether inside or outside. In relation to weather, the ambient temperature is the same as the current surrounding air temperature at any location. Ambient temperature can also be used to describe the state of objects.

2.1.2 Classifications of fluid flow

Laminar and Turbulence fluid flow: Laminar flow occurs when the fluid flows in infinitesimal parallel layers with no disruption between them. In laminar flows, fluid layers slide in parallel, with no eddies or currents normal to the flow itself. This type of flow is also referred to as streamline flow because it is characterised by non-crossing streamlines [4]. Turbulence flow is the type of flow in which the fluid particles move in a zigzag way, the eddies formation takes place which is responsible for high energy loss.

Viscous and Inviscid flow: Viscosity is the internal friction in fluid; viscous forces oppose the motion of one portion of a fluid relative to another [5]. It is said that when two fluid layers move relative to each other, frictional force develops between them which is quantified by the fluid property ‘viscosity’. Boundary layer flows are the example of viscous flow. Neglecting the viscous terms in the governing equation, the flow can be treated as an inviscid flow.

Steady and Unsteady fluid flow*:* Steady flow in the process, where by the velocity of a flowing fluid at a particular point does not change with time. The steady flow is also known as the stationary flow. A flow is also steady if the quantity of liquid flowing per second through any section is constant. Unsteady flow show temporal increases and decreases in velocity with time, which are often related to the passage of a discrete event? Unsteady flow is also the process whereby the velocity of a flowing fluid at a particular point changes with time.

Compressible flow and Incompressible flow: Compressible fluid flow is defined as the flow, in which the density of that fluid is not constant, which means the process whereby the density of a particular fluid changes from one point to another [11]. Compressible fluids are fluids with variable density, while incompressible fluids are fluids with constant density, constant volume. It could be liquid or gas.

2.1.3 Continuum hypothesis

Valuable deductions about behaviour and dynamics of fluid are obtained through the help of continuum hypothesis. In simple terms, when dealing with fluids we ignore the fact that fluid is built up or consist millions of molecules or atoms which are individually moving randomly in a rather small region. Instead we should treat the actual properties as if that small region is continuum.

The continuum hypothesis states that we can associate with any kinds of volume of fluid, no matter how small (but should be greater than zero) those macroscopic properties (e.g. **density**, **velocity**, **temperature** etc.) that we can always associates with the large fluid.

2.1.4 Conservation of mass (continuity equations)

In a close system like the flow system that is the focus of this project, conservation of mass states that the total mass of the system is constant. The following equations express the principle of conservation of mass,

$$\int \frac{\partial \rho}{\partial t}\, dv + \int \rho\vec{V} \cdot \mathrm{n}dA = \int \nabla \cdot (\rho\overrightarrow{V})\, dv$$

Therefore;

$$\int \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho\overrightarrow{V}) = 0 \qquad (2.2)$$

Or equally,

$$\frac{D\rho}{Dt} + \rho(\nabla \cdot \overrightarrow{V}) = 0 \qquad (2.3)$$

These equations are known to be the conservative and non-conservative forms of mass conservation, respectively. Where $\vec{V}$ is the velocity vector, $\boldsymbol{\rho}$ is the density, ∇ is the divergence, ***t*** is the time. Conservation forms of equations can be obtained by applying the physical principle (mass conservation in this case) to a fluid element in a fixed space. The Non-conservative forms are obtained by considering fluid elements moving in the flow field.

Therefore;

$$\frac{D\rho}{Dt} = \frac{\partial \rho}{\partial t} + \rho(\nabla \cdot \overrightarrow{V}) \qquad (2.4)$$

The link between these two equations can be established using the following general equation that relates spatial and material descriptions of fluid flow. Hence, let C be the material derivative or acceleration vector, then equation (3) becomes,

$$\frac{DC}{Dt} = \frac{\partial C}{\partial t} + C(\nabla \cdot \vec{V}) \qquad (2.5)$$

The term on the L.H.S of this equation is known as the material derivative of property (the moving element in flow field) and on the R.H.S is the partial time derivative or local derivative (the fluid element in a fixed space). The Last term is called the convective derivative of C.

2.1.5 Conservation of linear momentum (Newton's second law)

Equation for the conservation of linear momentum is also known as the Navier-Stokes equation. The term conservation of momentum which we are looking at states that the time rate of change of momentum of a system of particles is equal to the sum of external forces acting on that body. Nevertheless Navier-Stokes is usually used to include both momentum and continuity equations. It is possible to write it in many different forms. One possible form is;

$$\int F_B \, dV \qquad (2.6)$$

And;

$$\int F_S \, dA \qquad (2.7)$$

Note equation (5) and (6) are acting only on the surface S(t) and the body V(t), where the first equation is said to be the **body forces** acting on the entire region V(t), and the second equation is the **surface forces** S(t). It is very essential to view the surface S(t) as separating the fluid into two distinct regions: one that is interior to S(t), i.e., V(t), and the other on the outside of S(t). This implies that when we focus attention on V(t) alone, we must be responsible for the fact that we have omitted the outside—which interacts with V(t). We do this by representing these effects as surface forces acting on S(t).Therefore we treat both F_B and F_S, at the same time we produce the momentum equation which will leads us to the equation of motion using the Navier-Stokes theorem.

$$\int \frac{\partial \rho}{\partial t}\, dv = \int F_B \; dV \;\; + \int F_S \, d\mathrm{A} \qquad (2.8)$$

Therefore;

$$\frac{D}{Dt}\int \rho U \, dv = \int F_B \; dV \;\; + \int F_S \, d\mathrm{A} \qquad (2.9)$$

Where, $\frac{D}{Dt}\int \rho U \, dv$ is said to be the change in rate of momentum,

$$\frac{D}{Dt}\int \varphi \, dv = \int \frac{\partial \varphi}{\partial t} \; dV \;\; + \int \varphi U \, d\mathrm{A} \qquad (2.10)$$

Note that φ is a general vector field, but here we will work with only a single component at a time, so we can replace this with the scalar φ, and for the present discussions set $\varphi = \rho u$ component of momentum per unit volume. Substitution of this into (9),

$$\frac{D}{Dt}\int \rho u \, dv = \int \frac{\partial \rho u}{\partial t} \; dV \;\; + \int \rho u U \, n. d\mathrm{A} \qquad (2.11)$$

Then we apply gauss theorem to the surface integral (R.H.S), we have;

$$\int \frac{\partial \rho u}{\partial t} \; dV \;\; + \nabla \cdot (\rho u U) d\mathrm{A} \qquad (2.12)$$

Simplifying the R.H.S of equation (11) using product rule differential equation to obtain,

$$\nabla \cdot (\rho u) dA = U \cdot \nabla(\rho u) + \rho u \nabla \cdot \qquad (2.13)$$

We make use of the divergence-free condition of incompressible flow i.e $\nabla \cdot U = 0$ and constant density ρ, which give us;

$$\nabla \cdot (\rho u) dA = \rho U \cdot \nabla u \qquad (2.14)$$

Substituting into equation (11);

$$\frac{D}{Dt} \int \rho u \, dv = \int \frac{\partial \rho u}{\partial t} \, dV + \nabla \cdot (\rho u U) dA \qquad (2.15)$$

Therefore;

$$\int \rho \frac{Du}{Dt} dv \qquad (2.16)$$

We have used constant density and definition of the substantial derivative on the R.H.S. Thus, we have succeeded in interchanging differentiation (this time, total) with integration over the fluid element V (t). Therefore we impute our result in equation (15) into equation (7).

Our result becomes;

$$\int \rho \frac{Du}{Dt} dv = \int F_B \, dV + \int F_S \, dA \qquad (2.17)$$

2.1.6 Conservation of energy (First law of thermodynamics)

In section part we describe the energy of a moving element and how it changes from one point to another, we also show the derivation under the integral. Therefore, we say for any moving fluid, the region of volume M_t can define its energy as

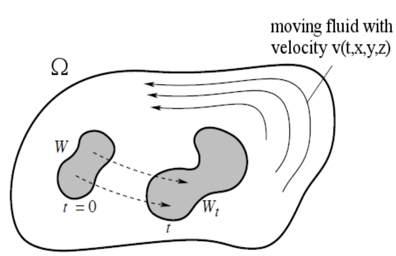
$$E_{M_t} = \int_{M_t} \rho \left(\frac{V^2}{2} + e\right) dV \,, \qquad (2.18)$$

Where ρ the density, v is the velocity field, and e is the density of the internal energy, M_t the set of points of the initial set (region) M displaced to a new location after time t. thus we have $M_0 = M_t$. Also;

$$E_{M_t} = \int_{M_t} \frac{\rho V^2}{2} dV = \text{The kinetic energy of the moving volume } M_t \qquad (2.19)$$

$$E_{int} = \int_{M_t} \rho e \, dV = \text{The internal energy of the moving volume } M_t \qquad (2.20)$$

Explain more about the conservation of energy, we are completely interested in how the quantity M_t changes with time when the fluid particle moves in the system.



Note that when moving the time derivation under the symbol of integral is not something very easy because it also affects the region of integration M_t which also depends on time. Therefore is not stationary!

Then we can rewrite the rate of change of time as;

$$\frac{d}{dt}\dot{\int}_{M_t} \rho\left(\frac{V^2}{2}+ e\right) dV = \dot{\int}_{M_t} \rho \frac{D}{Dt}\left(\frac{V^2}{2}+ e\right) dV \tag{2.21}$$

Applying the Raynolds transport theorem, we have;

$$\frac{d}{dt}\dot{\int}_{M_t} \rho f \, dV = \dot{\int}_{M_t} \rho \frac{Df}{Dt} \, dV \tag{2.22}$$

$\frac{Df}{Dt}$ is define as the material derivation, and for any quantity f(t,r) = f(t,x,y,z) that may depend on time and location and moving region M_t. To show the energy of a moving element M_t with other processes present in the system, this we lead us to the following general form of energy balance in the integral form.

$$\frac{d}{dt}\dot{\int}_{M_t} \rho\left(\frac{V^2}{2}+ e\right) dV = \dot{\int}_{M_t} \rho \mathbf{F} \cdot \mathbf{v} \, dV + \dot{\int}_{\alpha M_t} \rho \cdot \mathbf{v} \, dA + \dot{\int}_{\alpha M_t} (-J_Q) \cdot \mathbf{n} \, dA + \dot{\int}_{M_t} \rho R_q \, dV$$

Where; (2.23)

$\dot{\int}_{M_t} \rho \mathbf{F} \cdot \mathbf{v} \, dV$ = rate of work by **external** mass forces

$\dot{\int}_{\alpha M_t} \rho \cdot \mathbf{v} \, dA$ = rate of work by **internal** pressure forces

$\dot{\int}_{\alpha M_t} (-J_Q) \cdot \mathbf{n} \, dA$ = flux of the **heat** energy

$\dot{\int}_{M_t} \rho R_q \, dV$ = energy **gain/loss** due to the internal **sources/sinks** (e.g reaction)

$$\frac{d}{dt}\dot{\int}_{M_t} \rho\left(\frac{V^2}{2}+ e\right) dV = \dot{\int}_{M_t} \rho \mathbf{F} \cdot \mathbf{v} \, dV + \dot{\int}_{\alpha M_t} \rho \cdot \mathbf{v} \, dA + \dot{\int}_{\alpha M_t} (-J_Q) \cdot \mathbf{n} \, dA + \dot{\int}_{M_t} \rho R_q \, dV$$

Applying gauss theorem and to some part of the equation above in order to balance it completely to volume, therefore we have;

$$\int_{\alpha M_t}^{\cdot} \rho \cdot \mathbf{v}\ dA \ = \ \int_{M_t}^{\cdot} \nabla(\rho\mathbf{v})\, dV \tag{2.24}$$

$$\int_{\alpha M_t}^{\cdot} (-J_Q) \cdot \mathbf{n}\, dA = -\int_{M_t}^{\cdot} \nabla(J_Q)\ dV \tag{2.25}$$

Therefore we have;

$$\frac{d}{dt}\int_{M_t}^{\cdot} \rho\left(\frac{V^2}{2} + \ e\right) dV \ = \int_{M_t}^{\cdot} \rho\mathbf{F} \cdot \mathbf{v}\ dV + \int_{M_t}^{\cdot} \nabla(\rho\mathbf{v})\, dV \ - \int_{M_t}^{\cdot} \nabla(J_Q)\ dV \ \int_{M_t}^{\cdot} \rho R_q\ dV \tag{2.26}$$

Applying the transport theorem in equation (21) to the above equation, we have;

$$\int_{M_t}^{\cdot} \rho \frac{D}{Dt}\left(\frac{V^2}{2} + \ e\right) dV \ = \int_{M_t}^{\cdot} \rho\mathbf{F} \cdot \mathbf{v}\ dV + \int_{M_t}^{\cdot} \nabla(\rho\mathbf{v})\, dV \ - \int_{M_t}^{\cdot} \nabla(J_Q)\ dV \ \int_{M_t}^{\cdot} \rho R_q\ dV \tag{2.27}$$

Therefore the general equation for the conservation of energy is given as;

$$\rho \frac{D}{Dt}\left(\frac{V^2}{2} + \ e\right) \ = \rho\mathbf{F} \cdot \mathbf{v} + \nabla(\rho\mathbf{v}) - \ \nabla J_Q + \ \rho R_q$$

2.2 Divergence Theorem

The divergence theorem is typically the relationship between the surface integrals and the volume integrals, with the divergence of vector field which involved [8]. It often arises in mechanics problems, especially so in variation calculus problems in mechanics. The divergence theorem applied to a vector field **f** is;

$$\int_{v} \nabla \cdot \mathrm{f} dv = \int_{s} \mathrm{f} \cdot \mathrm{n} ds$$

The **LHS** is the volume integral over the volume, **V**, and the **RHS** is the surface integral over the surface enclosing the volume. **n.** the vector field, **f**, can be any vector field.

2.3 The Adomian Decomposition Method (ADM)

Adomian decomposition method is a method of solving both ordinary differential equation and partial differential equations. The method has gained an increasing research attention lately due to its effectiveness, especially in solving fluid dynamics equations. For example, Somali and Gokmen used Adomian decomposition method to solve the non linear sturm-lioville equation [6]. Adesanya and Ayeni also used the Adomian decomposition method to prove the existence of solution for a fluid dynamics equation [7]. Also, Kareem et el. used the Adomian decomposition method to solve an hydro magnetic couple stress fluid flow equations [1]. The method is especially preferred for this work because of its fast convergence and the avoidance of linearization in the solution process [8]. In the following subsection, we present the steps required in the solution of the ADM. These are presented in form examples.

2.4 Solving Partial Differential Equation using ADM

Here, we show how to obtain the ADM solution for partial differential equation (PDEs). First, we present the general procedure for solving PDE's. After this, we apply the procedure to obtain the solution of selected PDE's.

ADM is a series solution obtained by summing up some decomposed components of the unknown function, say **u(x, y),** of the equation to be solved. That is,

$$\mathbf{u[x, y]} = \sum_{n=0}^{\infty} u_n[x, y] \ldots\ldots\ldots\ldots 2.1$$

The aim here is to find the components $\boldsymbol{u_0}, \boldsymbol{u_1}, \boldsymbol{u_2}, \boldsymbol{u_3}, \ldots u_n$ individually. The equation to be solved may be given as

$$Lu + Ru = g \ldots\ldots\ldots\ldots 2.2$$

Where L is the order of derivative, R is linear differential operator, g is the source.

Then we apply the Inverse operator L^{-1} to equation (2)

$$L^{-1}(Lu + Ru = g)$$

$$u = f - L^{-1}(Ru) \ldots\ldots\ldots\ldots 2.3$$

$$\text{Where } u = \sum_{n=0}^{\infty} u_n$$

Therefore,

$$\sum_{n=0}^{\infty} u_n = u = f - L^{-1}\left(R\left(\sum_{n=0}^{\infty} u_n\right)\right) \ldots\ldots\ldots\ldots 2.4$$

For simplicity, equation (4) can be written as

$$u_0 + u_1 + u_2 + u_3 \ldots = f - L^{-1}(R(u_0 + u_1 + u_2 + u_3) \ldots (5)$$

Now, to determine the components (u_0 + u_1 + u_2 + u_3 ……), it is important to note that the ADM is always discovered by the function (f) as described above, also the inverse operator (L^{-1}) which arises from the initial data and also from integrating the homogenous term.

$$u_0 = f$$

$$u_{k+1} = -L^{-1} (R (u_k)) \text{ where } k \geq 0 \qquad \text{……………………………………2.5}$$

Therefore,

$$\left.\begin{aligned} u_0 &= f \\ u_1 &= -L^{-1} (R (u_0) \\ u_2 &= -L^{-1} (R (u_1) \\ u_3 &= -L^{-1} (R (u_2) \end{aligned}\right\} \qquad \text{…………………………………..2.6}$$

Let consider,

u'(x) = u(x), u(0) = A ………………………………………...............2.7

Imputing the operator,

Lu = u …………………………………………………………………...2.8

Differential operator:

$$L = \frac{d}{dx};$$

Therefore, inverse L^{-1} is defined as,

$$L^{-1} (.) = \int_0^x (.)\, dx \qquad \text{…………………………………………………………...2.9}$$

Applying L^{-1} to equation (x2),

$$L^{-1}(Lu) = L^{-1}(u)$$

$$L^{-1}(u) = u(x) - u(0)$$

$$u(x) = A + L^{-1}(u) \quad \ldots\ldots 2.10$$

Substituting equation (4) into equation (x4),

$$\sum_{n=0}^{\infty} u_n(x) = A + L^{-1}\left(\sum_{n=0}^{\infty} u_n(x)\right) \quad \ldots\ldots 2.11$$

Viewing recursive relation,

$$u_0(x) = A$$

$$u_{k+1}(x) = L^{-1}(R(u_k)) \text{ where } k \geq 0$$

Equivalently,

$$u_0(x) = A$$

$$u_1(x) = L^{-1}(u_0(x)) = Ax$$

$$u_2(x) = L^{-1}(u_1(x)) = \frac{Ax^2}{2!}$$

$$u_3(x) = L^{-1}(u_2(x)) = \frac{Ax^3}{3!} \quad \ldots\ldots 2.12$$

$$u(x) = A\left(1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots\ldots\right)$$

$$u(x) = Ae^x \quad \ldots\ldots 2.13$$

CHAPTER THREE

METHODOLOGY

3.0 Introduction

In this chapter, we present the model and the method of solution for Magneto hydrodynamic (MHD) fluid flow. First, the flow model is described. Usually, the equations obtained from the model are dimensional. In order to remove the dimensionality in the equations, we rescale or recast the equations into non-dimensional or dimensionless form. Therefore, after the description of the model, we obtain the dimensionless form of the flow equation. Thereafter, we present the Adomian decomposition method, which is applied to solve the flow equation. Lastly, we explain the implementation of the results from the application of the Adomian decomposition method in the symbolic package MATHEMATICA.

3.1 The Magneto hydrodynamic (MHD) Fluid Flow.

The model used in this project is the MHD fluid model. The model equations used in this project are the energy balance equation and the momentum balance equation [2]. For the purpose of this project, we consider a laminar flow, through a porous medium in a horizontal channel which is known to have impermeable walls, with the width $\boldsymbol{H}$. We imposed this channel with a magnetic field$\boldsymbol{B_0}$, which caused the fluid flowing through this medium to be electrically conducting$\boldsymbol{\sigma_0}$; we examine the model in a 2-dimentional Cartesian coordinate system, with x-axis along the horizontal and y-axis along the vertical axes. The horizontal channel is bounded by two rectangular plates and the flow direction is along the positive x-axis, with fluid velocity, u. Fig.3.1 shows the geometry of the mode [2]. The mathematical representation of the MHD model is given below in the form of the energy balance equation and the momentum balance equation:

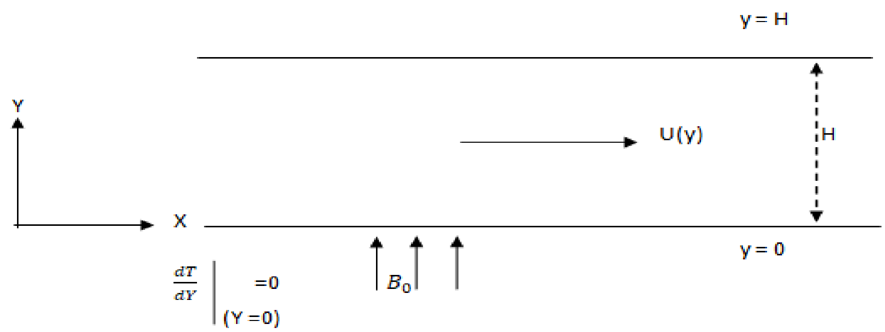


Fig. 3.1: The model geometry

The model equations are the momentum balance equations and the energy balance equations are given respectively as;

$$-\frac{\partial P}{\partial X} + \mu\frac{d^2U}{dY^2} - \mu\frac{U}{K^*} - \sigma_e B^2 U = 0 \qquad 3.1$$

And;

$$\rho c U\frac{\partial T}{\partial X} = K\frac{d^2T}{dY^2} + \mu(\frac{dU}{dY})^2 - \sigma_e B^2 U^2 = 0 \qquad 3.2$$

With the boundary condition:

$$u = 0, \frac{\partial T}{\partial Y} = 0, y = 0 \qquad 3.3$$

$$\mu\frac{\partial U}{\partial Y} = 0, T = T_1\, at\, Y = H, \qquad 3.4$$

The P is the pressure, μ is the kinematic viscosity, X and Y are the Cartesians coordinate, σ_e is the electric conductivity, B_0 is the magnetic field, ρ is the fluid density, T is the dimensional temperature, and k is the thermal conductivity.

The purpose of this project is to use the ADM to solve the equation of the model described above; that is equation Eqn (3.1) to (3.4). In order to do this, we first write the equations in the dimensionless form.

3.2 Dimensionless Form of the Model Equations

Firstly we solve for equation 3.1,

$$-\frac{\partial P}{\partial X}+\mu\frac{d^2U}{dY^2}-\mu\frac{U}{K^*}-\sigma_e B^2 U=0 \qquad 3.5$$

$$\text{Let } y=\frac{Y}{H}, u=\frac{y}{H}, \theta=\frac{T-T_0}{T_1-T_0} \qquad 3.6$$

Where v = characteristics velocity, μ = referenced dynamic fluid viscosity.

Since Y = Hy;

$$\frac{dY}{dy}=H \rightarrow \frac{dy}{dY}=\frac{1}{H}$$

$$\frac{d}{dY}=\frac{d}{dy}.\frac{dy}{dY}=\frac{1}{H}\frac{d}{dy} \rightarrow \frac{d^2}{dy^2}=\frac{1}{H^2}\frac{d^2}{dy^2} \qquad 3.7$$

Putting 3.6 and 3.7 into 3.5

$$-\frac{\partial P}{\partial X}+\frac{\mu}{H^2}\frac{d^2v_0u}{dy^2}-\mu\frac{v_0u}{K^*}-\sigma_e B^2 v_0 u=0 \qquad 3.8$$

$$-\frac{H^2}{\mu v_0}\frac{\partial P}{\partial X}+\frac{d^2u}{dy^2}-\frac{H^2u}{K^*}-\frac{\sigma_e B^2 H^2 u}{\mu}=0 \qquad 3.9$$

$$\frac{d^2u}{dy^2}=-G+\frac{H^2u}{K^*}+Mu \qquad 3.10$$

$$-G = \frac{H^2}{\mu v_0}\frac{\partial P}{\partial X}, \; M = \frac{\sigma_e B^2 H^2 u}{\mu} \text{ and let } K = \frac{H^2}{K^*}$$

. Therefore equation 3.5 becomes;

$$\frac{d^2u}{dy^2} = -G + Ku + Mu \qquad 3.11$$

Where K is the dimensionless porosity parameter, M is the dimensionless magnetic parameter and G is the dimensionless pressure gradient

Secondly, we solve for equation 3.2;

$$\rho c U \frac{\partial T}{\partial X} = K\frac{d^2T}{dY^2} + \mu(\frac{du}{dY})^2 + \sigma_e B^2 u^2 \qquad 3.12$$

Using equation 3.6, and nothing that;

$$\frac{d^2}{dY^2} = \frac{1}{H^2}\frac{d^2}{dy^2}, \qquad 3.13$$

$$\rho c U \frac{\partial T}{\partial X} = \frac{k}{H^2}\frac{d^2[(T_1 - T_0)\theta + T_0]}{dy^2} + \frac{\mu}{H^2}(\frac{d[v_0 u]}{dy})^2 + \sigma_e B^2 {v_0}^2 u^2 \qquad 3.14$$

$$\rho c v_0 U \frac{\partial T}{\partial X} = \frac{k[(T_1 - T_0)}{H^2}\frac{d^2\theta}{dy^2} + \frac{\mu {v_0}^2}{H^2}(\frac{du}{dy})^2 + \sigma_e B^2 {v_0}^2 u^2 \qquad 3.15$$

$$\frac{d^2\theta}{dy^2} = -\frac{H^2 \mu {v_0}^2}{H^2 k[(T_1 - T_0)}(\frac{du}{dy})^2 - \frac{\sigma_e B^2 H^2 {v_0}^2 u^2}{k[(T_1 - T_0)} + \frac{\rho c v_0 H^2 u}{k[(T_1 - T_0)}\frac{\partial T}{\partial X} \qquad 3.16$$

$$\frac{d^2\theta}{dy^2} = -\frac{\mu {v_0}^2}{k[(T_1 - T_0)}(\frac{du}{dy})^2 - \frac{\sigma_e B^2 H^2 {v_0}^2 u^2}{k[(T_1 - T_0)} + \frac{\rho c v_0 H^2 u}{k[(T_1 - T_0)}\frac{\partial T}{\partial X} \cdot u \qquad 3.17$$

Where;

$$B_r = -\frac{\mu {v_0}^2}{k[(T_1 - T_0)}, M = \frac{\sigma_e B^2 H^2}{\mu}, P_r = \frac{\rho c \vartheta}{k}, where\ \vartheta = \frac{\mu}{e} \qquad 3.18$$

$$\frac{d^2\theta}{dy^2} = -\frac{\mu {v_0}^2}{k[(T_1 - T_0)} (\frac{du}{dy})^2 - \frac{\sigma_e B^2 H^2}{\mu} \cdot \frac{\mu {v_0}^2 u^2}{k[(T_1 - T_0)} + \frac{\rho c \vartheta}{k} \frac{v_0 H^2}{\mu[(T_1 - T_0)} \frac{\partial T}{\partial X} \cdot u \qquad 3.20$$

Therefore;

$$\frac{d^2\theta}{dy^2} = P_r F u - B_r (\frac{du}{dy})^2 - M B_r u^2 \qquad 3.21$$

Where$F = \frac{v_0 H^2}{\mu[(T_1 - T_0)}$;

F is known as the dimensionless parameters, B_r is the Brickman's number, M is the dimensionless magnetic parameter, nevertheless equation 3.21 and 3.11 is said to be the dimensionless equation;

$$\frac{d^2 u}{dy^2} = -G + Ku + Mu$$

$$\frac{d^2\theta}{dy^2} = P_r F u - B_r (\frac{du}{dy})^2 - M B_r u^2$$

The dimensionless boundary condition is given as;

$$u(0) = 0, \frac{d\theta(0)}{dy} = 0, \frac{du(H)}{dy} = 0, \theta(H) = 1 \qquad 3.22$$

3.3 ADM Solutions

From equation (3.11), we impute the linear operator.

Therefore we have;

$$Lu = -G + Ku + Mu \qquad 3.23$$

Where $Lu = \frac{d^2}{dy^2}$ and the inverse is given as $L^{-1} = \int_0^y \int_0^y (.)dydy$, then we multiply both side of equation 3.23 byL^{-1}, therefore we have;

$$L^{-1}(Lu) = L^{-1}(-G) + L^{-1}(K + M)u \qquad 3.24$$

$$\frac{d^2u\ (y)}{dy^2} = (.)$$

$$\frac{du}{dy} = \frac{du(0)}{dy} + \int_0^y (.)dy$$

$$u(y) = u(0) + \int_0^y \frac{du(0)}{dy} dy + \int_0^y \int_0^y (.)dydy \qquad 3.25$$

Using equation 3.24 and 3.25,

$$u(y) = u(0) + \int_0^y \frac{du(0)}{dy} dy + L^{-1}(-G) + L^{-1}(K + M)u \qquad 3.26$$

From equation 3.23

$$u(0) = 0$$

$$u(y) = qy + L^{-1}(-G) + L^{-1}(K + M)u \qquad 3.27$$

For the recursive solution by ADM;

$$u(y) = \sum_0^n u_n(y)$$

$$u_0 = qy + L^{-1}(-G)$$

$$u_0 = qy + \int_0^y \int_0^y (-G)dydy$$

`

$$u_0 = qy - \frac{G}{2}y^2 \qquad 3.28$$

Then for (u), we have;

$$u_{k+1} = L^{-1}(K+M)u_k$$

$$u_1 = (K+M)\int_0^y \int_0^y (u_0)dydy$$

$$u_2 = (K+M)\int_0^y \int_0^y (u_1)dydy$$

$$u_3 = (K+M)\int_0^y \int_0^y (u_2)dydy$$

$$u_n = (K+M)\int_0^y \int_0^y (u_{n-1})dydy$$

Therefore u(y) is given as;

$$u(y) = \quad u_0(y) + \ u_1(y) + u_2(y) + u_3(y) + \cdots n(y) \qquad 3.29$$

Then we solving for equation (3.21) which is known as the second dimensionless equation using the ADM.

$$\frac{d^2\theta}{dy^2} = P_r F u - B_r (\frac{du}{dy})^2 - \ M B_r u^2$$

We write the equation the operator form,

$$L\theta = P_r Fu - B_r(\frac{du}{dy})^2 - MB_r u^2 \qquad 3.30$$

Where L = $\frac{d^2\theta}{dy^2}$

Inputting the inverse;

$$L^{-1}(L\theta) = L^{-1}[P_r Fu - B_r\left(\frac{du}{dy}\right)^2 - MB_r u^2] \qquad 3.31$$

$$L^{-1} = \int_0^{\theta}\int_0^{\theta}(.)\ dydy$$

Let $L\theta$(y) $= \frac{d^2\theta\ (y)}{dy^2}$ = (.)

=> $\frac{d\theta(y)}{dy} = \int_0^y \frac{d\theta(0)}{dy} dy + \int (.)$ dy

=> $\theta(y) = \theta(0) + \int_0^y \frac{d\theta(0)}{dy} dy + \int_0^{\theta}\int_0^{\theta}(.)$ dy dy 3.32

$$\theta(y) = \theta(0) + \int_0^y \frac{d\theta(0)}{dy} dy + L^{-1}P_r Fu - B_r(\frac{du}{dy})^2 - MB_r u^2 \qquad 3.33$$

$$\int_0^y \frac{d\theta(0)}{dy} dy = 0$$

$\theta(y) = g + L^{-1}\left[P_r Fu - B_r(\frac{du}{dy})^2 - MB_r u^2\right]$ dy dy 3.34

$\theta(y) = \theta_0(y) =$ g $+ L^{-1}\left[P_r Fu - B_r(\frac{du}{dy})^2 - MB_r u^2\right]$ dy dy 3.22

Where g$\theta_{k+1}(y) = 0$

Therefore;

$$\theta\ (y) = \theta_0(y)$$

In conclusion, the dimensionless equations eqn (3.11) and eqn (3.21), ADM solutions eqn (3.29) and eqn (3.23) together with the boundary conditions, eqn (3.22) where solved using the Adomain Decomposition Method and this algorithm was implemented using the software package- MATHEMATICA from which some dimensionless quantities like velocity and temperature were computed.

CHAPTER FOUR

RESULTS AND DISCUSSION

4.0 Introduction

The dimensionless equations Eqn (3.11) and Eqn (3.21) together with the boundary conditions, Eqn(3.22) obtained in the previous chapter were solved and plotted using the software package-MATHEMATICA. The results obtained are explained in this chapter. We start by explaining the effect of the magnetic field imposed on the channel, and how it affects the flow of the fluid, the velocity of the fluid and the temperature of the fluid.

4.1 Velocity Profile

In fig.4.1 the effect of magnetic field B_0 is shown on the velocity, with respect to the width of the flow of the channel. It is observed from this figure that as the magnetic field B_0 increases, the fluid velocity reduces. Consecutively, the flow velocity reduces. In fig.4.1 it is also observed that the flow rate of the fluid in the channel reduces due to the gradual increase in current generated from the magnetic field.

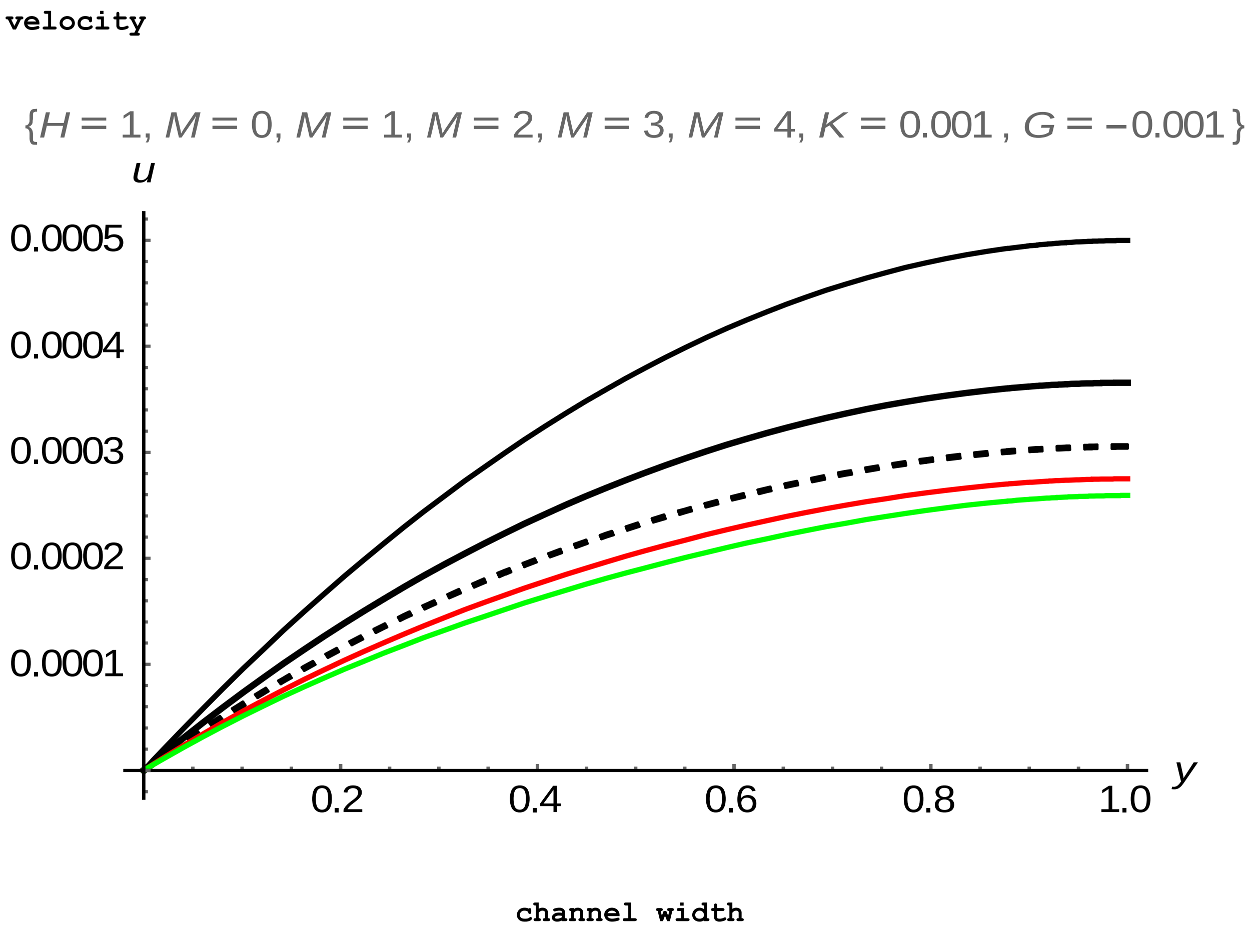


Figure 4.1 velocity profile of the fluid flow: M = 0, M = 1, M = 2, M = 3, M = 4, H = 1, K = 0.001, G = -0.001

4.2 Temperature Profile

In fig.4.2 the effect of magnetic field B_0 is shown on the temperature, with respect to the width H of the flow of the channel. It is observed from this figure that as the magnetic field B_0 increases, so as the fluid temperature increases. In fig.4.2 it is also observed that the temperature is constant at the width (H). In fig.4.2 it is also observed that the flow rate of the fluid temperature in the channel increases due to the gradual increase in joule heating generated from the magnetic field

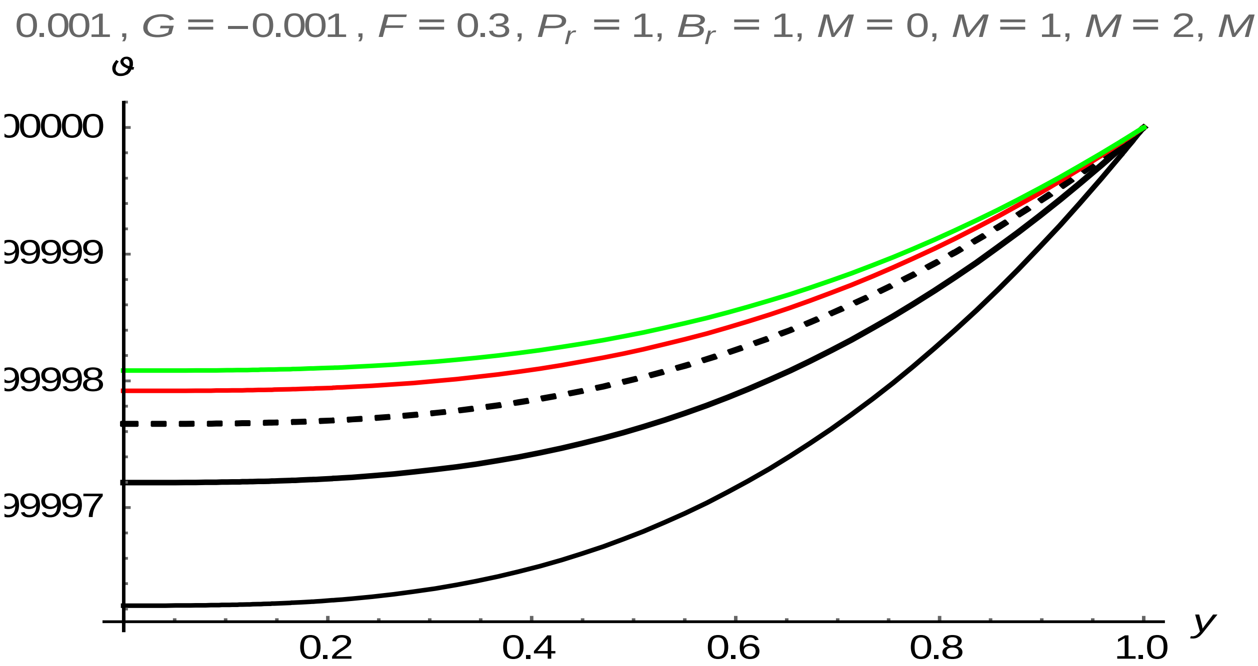


Figure 4.2 Temperature profile of the fluid flow: M = 0, M = 1, M = 2, M = 3, M = 4, H = 1, K = 0.001, G = 0.001, $P_r = 1, B_r = 1$

CHAPTER FIVE

CONCLUSION

5.1 Summary

This project examined the effect(s) of a laminar flow, through a porous medium in a horizontal channel under the influence of the imposed magnetic field, with the width ***H*.** The flow equations were non-dimensionalised equations, and solved by the Adomian Decomposition Method. The results shown are as follows;

1. There is a decrease in fluid velocity within the channel due to the increase in the imposed magnetic field.
2. There is an increase in fluid temperature within the channel due to the increase in the imposed magnetic field.

5.2 Recommendation

In this project, we discovered the fluid is electrically conducting which is due to the influence of the imposed magnetic field, with this alone it contributes to the Lorentz Force. Further research could be done to investigate the contribution of the electric conductivity to the Lorentz force from the external voltage.

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