PRAMANA — journal of physics © Indian Academy of Sciences Vol. 79, No. 1 July 2012 pp. 71–79

Function projective synchronization of identical and non-identical modified finance and Shimizu–Morioka systems

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MS received 11 November 2011; revised 4 January 2012; accepted 11 January 2012

Abstract. In this paper, function projective synchronizations (FPS) of identical and non-identical modified finance systems (MFS) and Shimizu–Morioka system (S-MS) are studied via active control technique. The technique is applied to construct a response system which synchronizes with a given drive system for a desired function relation between identical MFS, identical S-MS and between MFS and S-MS. The results are validated via numerical simulations.

Keywords. Function projective synchronization; modified finance system; Shimizu–Morioka system; active control technique.

PACS Nos 05.45.Pq; 05.45.Gg; 05.45.Ac

1. Introduction

Chaos synchronization is an important subject both theoretically and practically, for applications requiring oscillation out of chaos, machine and building structure analysis, chaos generator design and so on. Chaos synchronization first described by Fujisaka and Yamada [1] in 1983, did not receive much attention until 1990 [2]. From then on, chaos synchronization has developed extensively due to its various applications [3–5]. During the last decades, many techniques of handling chaos synchronization, such as Pecora and Carroll method [2], OGY method [6], feedback approach [7], adaptive method [8], time-delay feedback approach [9], backsteping design technique [10], sliding mode control [11], active control technique [12], etc. have been developed. Many of the methods mentioned above have been found effective for synchronizing two identical chaotic systems. But, it is well known that the components of most practical systems are non-identical. For instance, systems such as laser array, biological systems as well as cognitive processes consist of essentially non-identical components. Thus, it would be very instructive and significant to treat function projective synchronization of chaos in non-identical systems. This has been an open challenge that has recieved inadequate attention partly because non-identical systems have different dynamical structures as well as parameter mismatches.

Bai and Lonngren [13] proposed the method of identical chaos synchronization using active control. The technique was later generalized to non-identical systems by Ho and Hung [14], thus breaking the limit of its applicability beyond identical chaotic systems. Recently, the generalized active control (GAC) scheme [14] was employed by Chen and Lee to synchronize non-identical systems consisting of Lorenz, Chen and Lü systems with new chaotic systems attributed to Chen and Lee [15]. Chaos synchronization using active control has continued to receive wide application in a variety of dynamical systems such as geophysical model [16], spatiotemporal dynamical systems [17] etc.

The study of chaos synchronization has led to the discovery of various types of synchronization. These include complete synchronization [18], phase synchronization [19], lag synchronization [20], anticipating synchronization [20], projective synchronization [21], modified projective synchronization [22], function projective synchronization (FPS) [23], etc. In projective synchronization, the drive and the response systems synchronize up to a scaling factor whereas in modified projective synchronization, the response of the synchronized dynamical state variables synchronizes up to a constant matrix. Recently, a more general form of projective synchronization called function projective synchronization [24, 25] in which drive and response systems are synchronized up to a desired scaling function has attracted much attention of scientists and engineers as it provides more security in its applications to secure communication. Motivated by the above discussions, in this paper, we carried out FPS of identical and non-identical MFS and S-MS. The non-identical case is more interesting because the systems consist of different complex dynamical structures as well as parameter mismatches which can further enhance the security in secure communication. To our understanding, function projective synchronization of MFS and S-MS has not been explored.

The organization of the rest of this paper is as follows. Section 2 deals with system description. Section 3 deals with FPS between two identical MFS, between two identical S-MS evolving from different initial conditions, and between MFS and S-MS. In §4, we present numerical simulations to validate our results in §3, while the paper is concluded in §5.

2. System description

2.1 *Description of modified financial system*

The modified financial system is described as follows [21]:

$$\begin{aligned} \dot{x} &= z + (y - a)x + kx, \\ \dot{y} &= 1 - by - x^2, \\ \dot{z} &= -x - cz, \end{aligned} \tag{1}$$

where x, y and z are state variables and a, b, c and k are parameters. When $a = 0.6$, $b = 0.2$, $c = 0.9$ and $k = 0.5$, system (1) exhibits chaotic behaviour (figure 1).

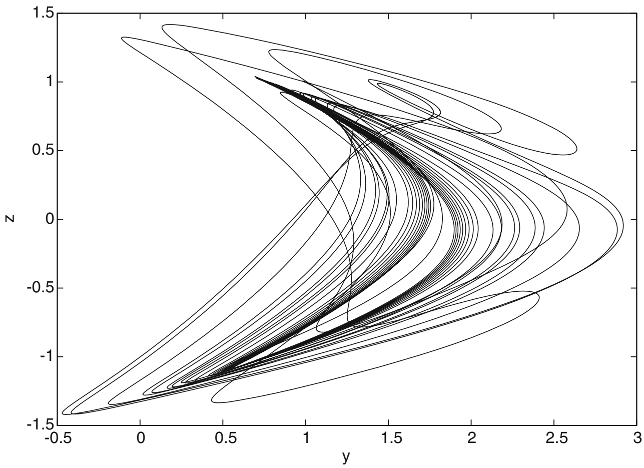

Figure 1. Phase space of the chaotic attractor portrait of the modified financial system for the parameter values $a = 0.6$, $b = 0.2$, $c = 0.9$ and $k = 0.5$.

2.2 *Description of Shimizu–Morioka system*

The modified Shimizu–Morioka system is described as follows [26]:

$$\begin{aligned} \dot{x} &= y, \\ \dot{y} &= x - \lambda y - xz, \\ \dot{z} &= -\beta z + x^2, \end{aligned} \tag{2}$$

where x, y and z are state variables and λ and β are parameters. When $\lambda = 0.605$ and $\alpha = 0.549$, system (2) exhibits chaotic behaviour (figure 2).

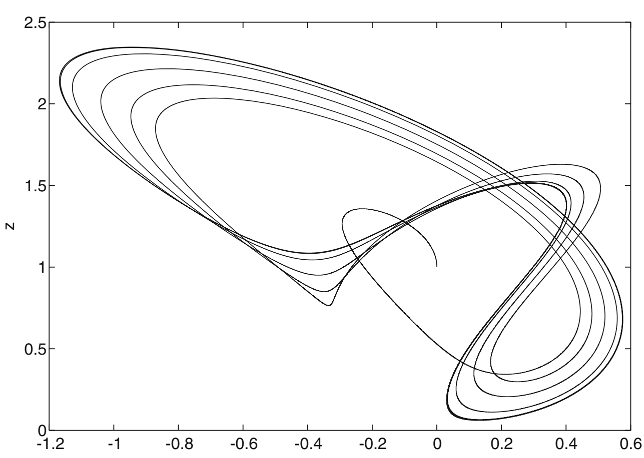

Figure 2. Phase space of the chaotic attractor portrait of the Shimizu–Morioka system for the parameter values $\alpha = 0.549$ and $\lambda = 0.605$.

3. Design of the synchronization scheme

3.1 *FPS between identical chaotic finance systems*

In this section, we present the FPS scheme for identical modified chaotic finance systems via active control scheme. The drive system is given as

$$\begin{aligned} \dot{x_1} &= x_3 + x_1(x_2 - a) + kx_1, \\ \dot{x_2} &= 1 - bx_2 - x_1^2, \\ \dot{x_3} &= -x_1 - cx_3, \end{aligned} \tag{3}$$

and the response system is given as

$$\begin{aligned} \dot{y_1} &= y_3 + y_1(y_2 - a) + ky_1 + u_1(t), \\ \dot{y_2} &= 1 - by_2 - y_1^2 + u_2(t), \\ \dot{y_3} &= -y_1 - cy_3 + u_3(t), \end{aligned} \tag{4}$$

where $u_i(t)$, $i = 1, 2, 3$ are the control functions to be determined. Our goal is to synchronize both drive and response systems to a scaling function α such that for the error states $e_i = y_i - \alpha x_i$, $\|e_i\| = 0$ as $t \to \infty$, where α is a time-dependent function. We obtain the error dynamics as follows:

$$\begin{aligned} \dot{e_1} &= e_3 + ke_1 - \dot{\alpha}x_1 + (y_1(y_2 - a) - \alpha x_1(x_2 - a)) + u_1(t) \\ \dot{e_2} &= (1 - \alpha) - be_2 - \dot{\alpha}x_2 - (y_1^2 - \alpha x_1^2) + u_2(t) \\ \dot{e_3} &= -e_1 - ce_3 - \dot{\alpha}x_3 + u_3(t). \end{aligned} \tag{5}$$

The control functions u_i's are re-defined to supress terms that are not linear in e_1, e_2 and e_3 as follows:

$$\begin{aligned} u_1(t) &= \dot{\alpha}x_1 - (y_1(y_2 - a) - \alpha x_1(x_2 - a)) + v_1(t) \\ u_2(t) &= -(1 - \alpha) + \dot{\alpha}x_2 + (y_1^2 - \alpha x_1^2) + v_2(2) \\ u_3(t) &= \dot{\alpha}x_3 + v_3(t) \end{aligned} \tag{6}$$

which gives the following matrix equation

$$\begin{pmatrix} \dot{e_1} \\ \dot{e_2} \\ \dot{e_3} \end{pmatrix} = \begin{pmatrix} k & 0 & 1 \\ 0 & -b & 0 \\ -1 & 0 & -c \end{pmatrix} \begin{pmatrix} e_1 \\ e_2 \\ e_3 \end{pmatrix} + \begin{pmatrix} v_1(t) \\ v_2(t) \\ v_3(t) \end{pmatrix} \tag{7}$$

where

$$\begin{pmatrix} v_1(t) \\ v_2(t) \\ v_3(t) \end{pmatrix} = A \begin{pmatrix} e_1 \\ e_2 \\ e_3 \end{pmatrix}. \tag{8}$$

A is a 3×3 matrix defined as

$$A = \begin{pmatrix} \lambda_1 - k & 0 & -1 \\ 0 & \lambda_2 + b & 0 \\ 1 & 0 & \lambda_3 + c \end{pmatrix}, \qquad \lambda_i \ (i = 1, 2, 3) < 0. \tag{9}$$

3.2 *FPS between identical Shimizu–Morioka systems*

The drive of the Shimizu–Morioka system is written as

$$\begin{aligned}\dot{x_1} &= x_2\\ \dot{x_2} &= x_1 - \lambda x_2 - x_1 x_3\\ \dot{x_3} &= -\beta x_3 + x_1^2\end{aligned} \tag{10}$$

and the response system is given as

$$\begin{aligned}\dot{y_1} &= y_2 + u_1(t)\\ \dot{y_2} &= y_1 - \lambda y_2 - y_1 y_3 + u_2(t)\\ \dot{y_3} &= -\beta y_3 + y_1^2 + u_3(t).\end{aligned} \tag{11}$$

As in §3.1, the control functions u_i's added to the response system ensure the synchronization of both the drive and the response systems with the error functions defined as $e_i = x_i - \alpha y_i$ $(i = 1, 2, 3)$

$$\begin{aligned}\dot{e_1} &= e_2 - \alpha u_1(t) - \dot{\alpha} y_1\\ \dot{e_2} &= e_1 - \lambda e_2 - x_1 x_3 + \alpha y_1 y_3 - \alpha u_2(t) - \dot{\alpha} y_2\\ \dot{e_3} &= -\beta e_3 + x_1^2 - \alpha y_1^2 - \alpha u_3(t) - \dot{\alpha} y_3.\end{aligned} \tag{12}$$

We redefine the control functions to eliminate functions that are not linear in terms of e_1, e_2 and e_3

$$\begin{aligned}u_1(t) &= 1/\alpha(-\dot{\alpha} y_1 + v_1(t))\\ u_2(t) &= 1/\alpha(-x_1 x_3 + \alpha y_1 y_3 - \dot{\alpha} y_2 + v_2(t))\\ u_3(t) &= 1/\alpha(-x_1^2 - \alpha y_1^2 - \dot{\alpha} y_3 + v_3(t)).\end{aligned} \tag{13}$$

We now obtained the error dynamics as

$$\begin{pmatrix}\dot{e_1}\\ \dot{e_2}\\ \dot{e_3}\end{pmatrix} = \begin{pmatrix}0 & 1 & 0\\ 1 & -\lambda & 0\\ 0 & 0 & -\beta\end{pmatrix}\begin{pmatrix}e_1\\ e_2\\ e_3\end{pmatrix} + \begin{pmatrix}v_1(t)\\ v_2(t)\\ v_3(t)\end{pmatrix}, \tag{14}$$

where

$$\begin{pmatrix}v_1(t)\\ v_2(t)\\ v_3(t)\end{pmatrix} = M\begin{pmatrix}e_1\\ e_2\\ e_3\end{pmatrix}. \tag{15}$$

M is a 3×3 matrix defined as

$$M = \begin{pmatrix}-\lambda_1 & 1 & 0\\ 1 & -\lambda - \lambda_2 & 0\\ 0 & 0 & -\lambda_3 - \beta\end{pmatrix}, \qquad \lambda_i\ (i = 1, 2, 3) < 0. \tag{16}$$

3.3 *FPS between non-identical chaotic finance system and Shimizu–Morioka system*

This subsection deals with FPS synchronization between MFS and S-MS. Let the drive system be the MFS (3) and the response system be the S-MS (11). We define the error states as $e_i = y_i - \alpha x_i$ and the error dynamics system is given as

$$\begin{aligned}
\dot{e_1} &= y_1 - \dot{\alpha} x_1 - \alpha(x_3 + x_1(x_2 - a) + kx_1) + u_1(t) \\
\dot{e_2} &= y_1 - \lambda y_2 - y_1 y_3 - \dot{\alpha} x_2 - \alpha(1 - bx_2 - x_1^2) + u_2(t) \\
\dot{e_3} &= -\beta y_2 + y_1^2 - \dot{\alpha} x_3 - \alpha(-x_1 - cx_3) + u_3(t).
\end{aligned} \tag{17}$$

Again, the control functions are defined to eliminate non-linear terms in e_1, e_2 and e_3 as follows:

$$\begin{aligned}
u_1(t) &= -y_2 + \dot{\alpha} x_1 + \alpha(x_3 + x_1(x_2 - a) + kx_1) + v_1(t)) \\
u_2(t) &= -y_1 + \lambda y_2 + y_1 y_3 + \dot{\alpha} x_2 + \alpha(1 - bx_2 - x_1^2) + v_2(t)) \\
u_3(t) &= -\beta y_3 - y_1^2 + \dot{\alpha} x_3 + \alpha(-x_1 - cx_3) + v_3(t)).
\end{aligned} \tag{18}$$

The error dynamics system is then redefined as

$$\begin{pmatrix} \dot{e_1} \\ \dot{e_2} \\ \dot{e_3} \end{pmatrix} = P \begin{pmatrix} e_1 \\ e_2 \\ e_3 \end{pmatrix} + \begin{pmatrix} v_1(t) \\ v_2(t) \\ v_3(t) \end{pmatrix}, \tag{19}$$

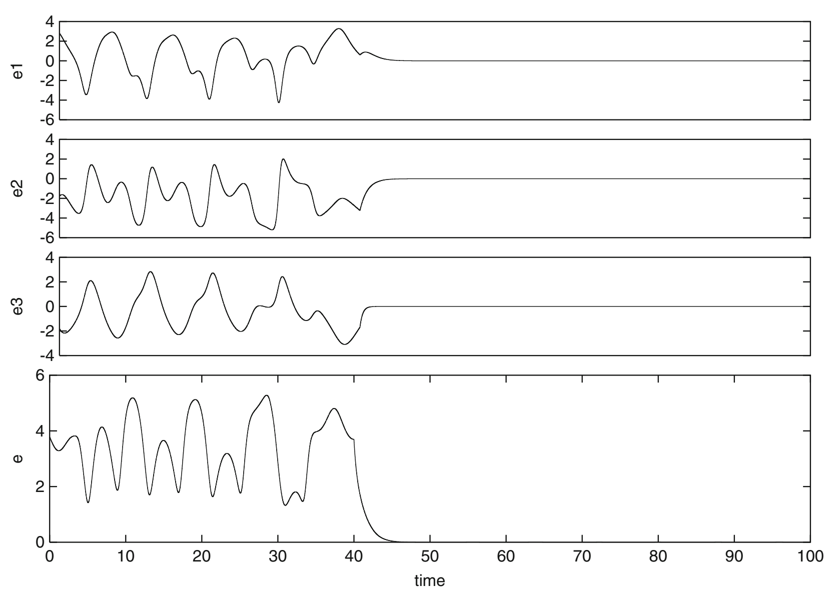

Figure 3. Error dynamics between two modified financial systems with the controllers deactivated for $0 < t < 40$ and activated for $t \geq 40$ where the scaling function $f(t) = 2 + 0.3\cos 2t$.

where P is a 3×3 null matrix and

$$\begin{pmatrix} v_1(t) \\ v_2(t) \\ v_3(t) \end{pmatrix} = N \begin{pmatrix} e_1 \\ e_2 \\ e_3 \end{pmatrix}, \tag{20}$$

where

$$N = \begin{pmatrix} -\lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{pmatrix}, \qquad \lambda_i \ (i = 1, 2, 3) < 0. \tag{21}$$

4. Numerical simulations

To further show the correctness of our procedures in §3, we employed numerical integration using the fourth-order Runge–Kutta method to solve each synchronization formulation. Figure 3 shows that the synchronization problem in §3.1 has been effectively solved. Here we plotted the time series of the errors e_1, e_2 and e_3, as well as the average error e given by $\langle e\rangle = (\sum e_i)^{1/2}$, where $i = \{1, 2, 3\}$. The states' discrepancies between systems (3) and (4) vanished as the time-dependent controllers (6) were activated at time $t \geq 40$ units. For this, the scalling function is $\alpha = 2 + 0.3\cos 2t$. Figure 4 shows the effectiveness of the synchronization scheme in §3.2. Again, by activating the time-dependent

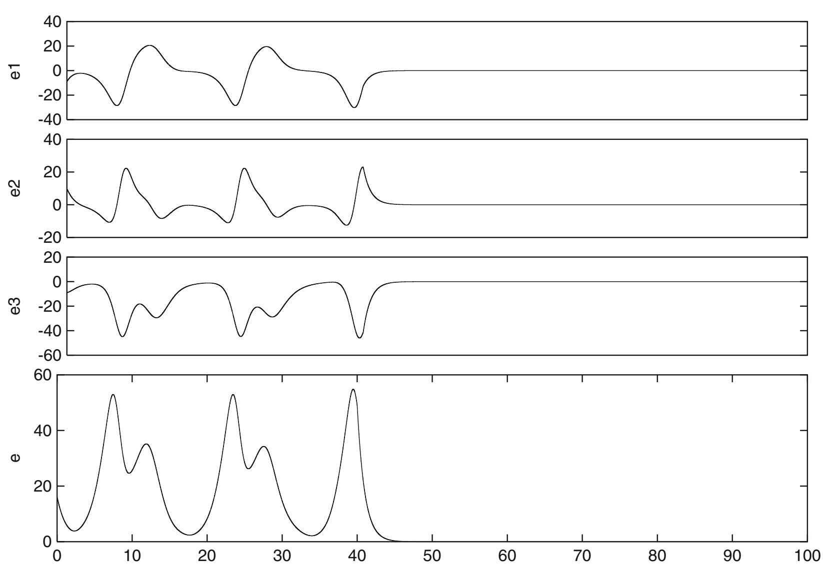

Figure 4. Error dynamics between two Shimizu–Morioka systems with the controllers deactivated for $0 < t < 40$ and activated for $t \geq 40$ where the scaling function $f(t) = 20 + \sin 0.02t$.

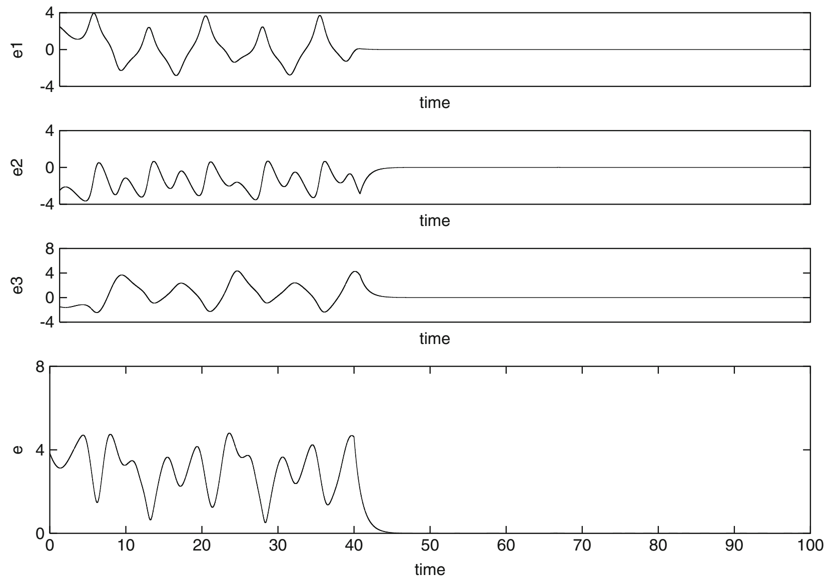

Figure 5. Error dynamics between modified financial system and the Shimizu–Morioka systems with the controllers deactivated for $0 < t < 40$ and activated for $t \geq 40$ where the scaling function $f(t) = 2 + 0.1\sin(0.1\pi t/40)$.

controllers (13) at time $t \geq 40$ units, the error dynamics between the Shimizu–Morioka systems (10) and (11) for $\alpha = 20 + \sin 0.02t$ tends to zero. Figure 5 shows the effectiveness of the synchronization scheme in §3.3 wherein the time-dependent controllers (18) were activated at $t \geq 40$ units with $\alpha = 2 + 0.1\sin(0.1\pi/40)t$. As shown in the figure, the error states converged to zero as soon as the controllers were activated, thereby guaranteeing FPS between the MFS and S-MS.

5. Conclusion

This paper describes chaos synchronization in identical and non-identical modified finance and Shimizu–Morioka systems using the FPS via active control method. With the help of numerical simulations, it has been shown that the FPS method can guarantee stable synchrony between these systems.

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