VECTORS, GEOMETRY AND DYNAMICS

COURSE CODE: MTH104

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Course Contents

References

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Chapter 1

Straight Line

1.1. Distance Between two points

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Example 1

Find the distance between the following point

i. $(3, -2)$ and $(4,\ 6)$
ii. $(-3,\ 2)$ and $(4,\ 5)$

Example 2

The distance between the points $(4,\ P)$ and $(1,\ 1)$ is 5 find the possible value of p.

Example 3

Show that the points $(2,\ 1), (4,\ 0)$ and $(5,\ 7)$ form vertices of a right angled triangle

1.2. Polar and Cartesian Coordinate

The polar form of the coordinate $(x,\ y)$ is (r, θ)

where

$$r^2 = x^2 + y^2$$

$$\theta = \tan^{-1}(y/x)$$

$$y = r \sin \theta$$

$$x = r \cos \theta$$

Example 3

What is the Cartesian coordinate of the points whose polar coordiantes are $(7,\ \frac{\pi}{2})$

Example 4

Find the polar coordinate of the point whose Cartesian coordinate is $(-5,\ 5)$

1.3. Division of a Line Segment Internally

The point of division of a line segment internally in the ratio $a : b$ is given as

$$x = \frac{bx_1 + ax_2}{a+b}, \qquad y = \frac{by_1 + ay_2}{a+b}$$

Thus,

$$(x, y) = \left(\frac{bx_1 + ax_2}{a+b}, \qquad \frac{by_1 + ay_2}{a+b}\right)$$

1.4. Mid-Point

In the case of mid-point, the ratio $a : b\ is\ 1 : 1$

Thus,

$$x = \frac{x_1 + x_2}{2}, \qquad y = \frac{y_1 + y_2}{2}$$

Hence,

The mid-point of the points $(x_1, y_1)\ and\ (x_2, y_2)$ is

$$\left(\frac{x_1 + x_2}{2}, \qquad \frac{y_1 + y_2}{2}\right)$$

Example 5

Find the coordinates of the point which divides the line joining the points $(-3,6)\ and\ (7,10)$ internally in the ratio $5 : 1$

Example 6

Find the mid-point of the line segment joining the points $(6,4)\ and\ (8,5)$

Example 7

Point A is $(11,1)$ and point B is $(2,6)$. Point P lies on the line AB and $2AP = PB$. Find the coordinates of p

1.5. Division of a Line Segment Externally

The coordinates of the point which divides the line segment externally in the ratio $a : b$ is given as

$$(x, y) = \left(\frac{bx_1 - ax_2}{b - a}, \quad \frac{by_1 - ay_2}{b - a}\right)$$

Example 8

Find the coordinates of the point which divides the line passing the points $(-2,4), (0,3)$ in the ratio $1 : 2$

1.6. Gradient of a Line Segment

Suppose $P(x_1, y_1)$ and $Q(x_2, y_2)$ are two points on a plane, let the line joining P and Q be PQ. Then the gradient or slope of the straight line PQ is given as

$$m = \frac{change\ in\ y}{change\ in\ x} = \frac{y_2 - y_1}{x_2 - x_1}$$

From trigonometry $\triangle PQR$ is the right angled triangle

$$\tan\theta = \frac{opposite}{adjacent} = \frac{y_2 - y_1}{x_2 - x_1}$$

Hence, the gradient of a line is the tangent of the angle the line makes with the positive $x - axis$

1.6.1 Relationship between Gradients of Parallel and Perpendicular Lines

Parallel line has equal gradient.

While for two lines that are perpendicular, the product of their gradient is -1

Example 9

Given the following coordinates, check whether the two lines are parallel or perpendicular $[M(12,14)\ N(22, 6)]$ and $[O(-4, 10),\ P(4, 20)]$.

Example 10

Find the gradient of the line joining the point $P(4,\ 10)\ and\ Q(8,\ 18)$.

Example 11

What are the gradients of the lines joining the origin to the points of intersection of

$$y = x^2\ \ and\ 2y = x + 1.$$

1.7. Straight Line Equations

Another name for straight line equation is linear equation.

1.7.1 Equation of a Straight line through a given point

$$y - mx = c\ \ or\ \ y = mx + c$$

The above equation is referred to as the “slope-intercept” form. The intercept c is on the y-axis

Example 12

Find the equation of the straight line that has gradient $m = 4$ and passes through the point $(-2, -10)$.

1.8.