# ENTROPY GENERATION IN A MAGNETOHYDRODYNAMIC FLUID FLOW THROUGH A VERTICAL DEFORMABLE POROUS MEDIUM


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**IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR THE AWARD OF BACHELOR OF SCIENCE DEGREE IN PHYSICS.**


**SEPTEMBER, 2022.**

## DECLARATION

I hereby declare that this project report written under the supervision of Dr. S. O. Kareem is a product of my own research work. Information derived from various sources has been duly acknowledged in the text and a list of references provided. This research project report has not been previously presented anywhere for the award of any degree or certificate.

ISHIEGBU, MOSES CHIDERA

Date

## CERTIFICATION

This is to certify that the content of this project entitled “ENTROPY GENERATION IN AN MHD FLUID FLOW THROUGH A VERTICAL DEFORMABLE POROUS MEDIUM” was prepared and submitted by ISHIEGBU, Moses Chidera with matriculation number 18010302001, in partial fulfillment of the requirements for the award of the degree of Bachelor of Science in Physics, Department of Physics of Mountain Top University, Ogun State, Nigeria. The original research work was carried out by him under my supervision and is hereby accepted.

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## DEDICATION

I dedicate this project work first and foremost to God Almighty for making this project work a success. I would also like to dedicate it to my parents, Mr. and Mrs. Fidelis Kanikwu-Ishiegbu for their everlasting love and support.

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## NOMENCLATURE

v: fluid velocity

Br: Brinkman number

x,y: Cartesian coordinates

$\rho$: density

$G_r$: Grashof number

$N_S$: entropy generation number

$Q_o$: heat source

M: magnetic parameter

u: solid displacement

$\theta$: temperature

$\mu_a$: apparent viscosity of fluid in porous material

$\mu_f$: coefficient of viscosity

K: drag coefficient

$\mu$: Lame constant

η: ratio of bulk fluid viscosity to apparent fluid viscosity in porous layer

$K_o$: thermal conductivity

$\varphi$: volume fraction of the fluid

$v$: velocity of the fluid

$\delta$: viscous drag

h: width of the plates

$T_0$: plate temperature at y = 0

$T_w$: temperature of plate at y = h

## ABSTRACT

This project investigates entropy generation in a magnetohydrodynamic (MHD) fluid flow through a vertical deformable porous medium. The model equations for the solid displacement, velocity, temperature and entropy generation number governing the fluid flow in the porous medium in their dimensional form were converted to non-dimensional form. The Adomian Decomposition Method (ADM) was applied to obtain the recursive scheme for each of the non-dimensional equations. The recursive scheme obtained solves the non-dimensional differential equations. The package MATHEMATICA was used to implement the scheme. The solutions obtained were also represented graphically to further explain the behaviour of the MHD fluid with varying conditions. The results obtained showed that entropy generation increases with increase in the viscous dissipation parameter.

# CHAPTER ONE

# INTRODUCTION

## 1.0 Background to the study

The concept of viscous flow of a fluid through porous media in fluid mechanics has been heavily investigated and researched for decades and has subsequently led to various applications in some fields such as that of geology as well as medicine, with (Terzaghi, 1925) being the first among others to initiate the study of fluid flow through deformable porous media. Afterwards, (Biot, 1962) posited a mathematical model for the acoustic propagation and the deformation mechanics of fluid flow in porous media. Thereafter, a series of closely-related researches followed such as the theory of immiscible and structured mixtures conducted by a few researchers such as (Atkin and Craine, 1976), (Bown, 1980) and (Bedford and Drumheller, 1983). (Jayaraman, 1983) then studied the transport of water through the arterial wall and then (Jain and Jayaraman, 1987) analysed a theoretical model for water flux through an arterial wall, after which more research followed and then (Sreenadh *et al.*, 2018) published a paper where the analysis for entropy generation for a magnetohydrodynamic(MHD) fluid flowing through a deformable porous layer was investigated.

MHD is a key aspect of fluid mechanics and can be defined as the analysis of the behaviour of the dynamics and the magnetic properties of fluids that are electrically conducting (Sheikholeslami and Ganji, 2016), and is the basis for which the research for this project work was conducted.

## 1.1 Statement of the Problem

The behaviour of the viscous flow of MHD fluid in porous media can at times be unpredictable and due to the effect of the fluid's internal friction on the flow of the fluid and with the presence of the magnetic field that could be imposed on the fluid flow.

## 1.2 Aim and objectives of the study

The aim of the project is to study the effects of the flow related to physical parameters on the entropy generation in an MHD fluid flowing through a deformable porous media with certain boundary conditions.

The specific objectives of this project are to;

i. Convert the model equation to non-dimensional form.
ii. Apply the boundary conditions to the equations and hence find the solution of the equations using Adomian Decomposition Method(ADM).
iii. Implement the solutions obtained from the ADM using MATHEMATICA, a mathematical software that has various applications, to determine the behaviour of the fluid subject to different conditions.
iv. Use Mathemathica to generate graphs for the solid displacement, fluid velocity, fluid temperature and the entropy temperature.

## 1.3 Significance of the Study

This project will further investigate behavioural changes of physical variables of an MHD fluid under certain conditions and the effect it has on the fluid flow. The results can be applied in the industry to make engineering decisions in the production of fluid related devices.

## 1.4 Project Outline

The first chapter introduces the project, the aims and objectives of the project, the significance of the project work and its relevance to the world at large. The second chapter however contains the literature relevant to the project, where the concept of fluid mechanics and the related equations were reviewed. Chapter three contains the methodology used in the project, and the results and discussions were presented in chapter four. Chapter five contains the conclusion and recommendations from the project.

# CHAPTER TWO

# LITERATURE REVIEW

## 2.0 Introduction

Fluid mechanics, an interesting field of study in the world of science with its diverse applications is a phenomenon that has come to stay and can only expand with more and more research done with each research bringing in one discovery or the other. To have a better understanding of the magnetohydrodynamics(MHD) and aid in the research of this project, we will delve into fluids in general, types of fluids, properties of fluids, viscosity of fluids, the various laws of thermodynamics, and the different kinds of fluid flow in this chapter.

## 2.1 Fluid

Primarily, there are five different states of matter that categorizes one of the distinguishable forms that matter can exist namely, solids, liquids, gases, plasma and Bose-Einstein condensate. Fluids are substances (liquids or gases) having no definite shape that flow or conform to the existing boundaries of the empty container in which they occupy. The field of fluid mechanics has diverse applications in various fields including mechanical engineering, civil engineering, biology, geophysics, chemical engineering, oceanography, biomedical engineering, amongst several others. It can be further subdivided into fluid statics and fluid dynamics.

### 2.1.1 Fluid Statics

This is the branch of fluid mechanics that studies fluids at rest and it can also be known as hydrostatics. It is only concerned with the study of the circumstances in which fluids are at rest and are in a stable equilibrium. Hydrostatics provides physical justifications for a variety of daily phenomena, including why air pressure varies with height, why oil and wood float, and why water's surface remains consistently level regardless of the shape of its container. Its application is relevant to many subjects, including meteorology, medicine (with regards to blood pressure), and some parts of geophysics and astronomy (such as, in understanding anomalies and plate tectonics in the gravitational field of planet Earth).

### 2.1.2 Fluid Dynamics

It is a branch of fluid mechanics that deals solely with fluid flow. Usually, problem-solving in fluid dynamics involves the calculation of various physical properties of the fluid, such as the temperature, velocity, pressure, and density. It has two sub-disciplines, such as hydrodynamics, the study of liquids that are said to be in motion and aerodynamics, the study of the way different objects move in air and other types of gases that are said to be in motion. Fluid dynamics also has diverse applications, such as the calculation of movements and forces on aircrafts, the determination of the mass flow rate of the movement of petroleum through pipelines and the prediction of changes in weather pattern (Wikipedia).

## 2.2 Properties of Fluids

Fluids have a number of variables that aid in defining and understanding the properties of the fluid whether it be the kinematic property, thermodynamic property or physical property. The kinematic property aids in understanding the motion of the fluid and is defined by the velocity and the acceleration of the fluid. The thermodynamic property aids in understanding the thermodynamic state of the fluid. Thermodynamic properties of fluids include temperature, density and pressure.

### 2.2.1 Density

**Mass Density:** The density of a material or substance can be defined as the mass per unit volume of that material (Jones, 2020). It has the unit of $kg/m^3$ and is represented by the Greek letter, $\rho$ (rho). Mass Density,

$$\rho = \frac{m}{V}, \qquad 2.1$$

where; m is the mass of the substance and v is the volume of the substance.

**Relative Density:** It can be defined as the ratio of the density of a material to the density of a given material or substance of reference.

Relative density, $SG = \frac{\rho_m}{\rho_{rm}}$. 2.2

Where $\rho_m$ is the density of the material and $\rho_{rm}$ is the density of the reference material. It has a unit of $N/m^3$.

### 2.2.2 Pressure

It can be defined as the force per unit area applied perpendicularly to the surface of an object. It is usually represented as

$p = \frac{F}{A}$, 2.3

where F is the force and A is the area. It has an S.I. unit of Pa(pascal).

### 2.2.3 Temperature

It can be described as the physical quantity that measures the hotness/coldness or matter of radiation of a system using any number of arbitrary scales and showing the direction of the spontaneous flow of the heat energy(Britannica). Temperature normally indicates what direction in which heat energy will flow (i.e. from a substance with a higher temperature to a substance of lower temperature(Britannica) and it has an SI unit of K (kelvin).

### 2.2.4 The Laws of Thermodynamics

The history of thermodynamics has its roots in ancient conceptions of heat and is inextricably entwined with the histories of physics and chemistry. The development in this area during the late nineteenth and early twentieth century led to the development of the laws of thermodynamics. (Carnot, 1824) wrote a book titled *Réflexions sur la puissance motrice du feu et sur les machines propres à développer cette puissance* (which translates to "Reflections on the Motive Power of Fire" in english), which contained the first accepted thermodynamic concept, which later developed into the second law of thermodynamics. Walther Nernst later developed the third law of thermodynamics, often known as Nernst's theorem (or Nernst's postulate), between 1906 and 1912. Although the numbering of the laws is now universal, different textbooks during the 20th century assigned different numbers to the laws. In some domains, the second law was thought to exclusively apply to the effectiveness of heat engines, whilst the third law was thought to apply to increases in entropy. This eventually resolved itself and later, the zeroth law was added to allow for a definition of temperature that is self-consistent. Although more laws have been proposed, none of them have attained the same level of generality as the four recognised laws, and thus are typically not covered in required textbooks.

**2.2.4.1 The Zeroth Law of Thermodynamics**

The zeroth law of thermodynamics establishes the transitive link between the temperatures of various entities in thermal equilibrium and lays the groundwork for temperature as a pragmatic specification in thermodynamic systems. It states that if two systems are in thermal equilibrium with a third system, then they are said to all be in thermal equilibrium with one another (Buchdahl, 1966).

**2.2.4.2 The First Law of Thermodynamics**

The first law of thermodynamics is a thermodynamic adaptation of the law of conservation of energy. According to the law of conservation, energy cannot be created, nor can it be destroyed but it can be transformed from one form to another, keeping the total energy of an isolated system constant.

It states that in a closed system, the difference between the heat that's supplied into the system and the work done by the system determines the change in the internal energy of the system.

$$U_{system} = Q - W \qquad 2.4$$

Where $U_{system}$ is the change in the internal energy of the system, $Q$ is the heat that's supplied into the system and $W$ is the work done by the system.

#### 2.2.4.3 The Second Law of Thermodynamics

The second law of thermodynamics highlights the irreversibility of natural processes and, frequently, the tendency for natural processes to result in spatial homogeneity of matter and energy, particularly temperature. It can be put forth in a number of intriguing and significant ways. The Clausius assertion that heat does not naturally transfer from a colder to a hotter body is one of the most basic.

When two systems, that were initially isolated and were each in their states of thermodynamic equilibrium, are separated but are in a close region of space and are allowed to interact with one another they will reach a thermodynamic equilibrium that's said to be mutual. It is said to be mutual because the entropy of the combination of the two systems will be greater than or equal to the sum of the entropies of the isolated systems that were initially isolated. Basically, the law explains that the entropy of an isolated system will never decrease over time.

#### 2.2.4.4 The Third Law of Thermodynamics

It states that when the temperature of a system approaches absolute zero (-273K), the entropy of the system would approach a constant value.

### 2.2.5 Viscosity

It can be described as the measure of the resistance to a fluid flow due to the internal friction of the fluid. Generally, a fluid's viscosity depends on the number of properties/states of the fluid such as the rate of deformation of the fluid, the temperature and the pressure of the fluid. There are two different types of viscosity: dynamic and kinematic viscosity. Kinematic viscosity can be described as the measure of the resistive fluid flow under the influence of gravity, whiles dynamic viscosity can be described as the measure of the resistance to shearing flows of a fluid, in which adjacent layers move parallel to one another at different speeds.

Consider two plates that are y distances apart, separated by a homogeneous substance, to obtain the relationship between the shear stress and velocity gradient. Assuming that the plates have a big area A and are very large, it is possible to ignore edge effects. Also, assuming that the lower plate is fixed, a force, F, will be applied to the top plate. The substance between the plates is said to behave as a fluid if this force induces shear flow with a velocity gradient, u, instead of only elastic shearing until the applied force is balanced by the shear stress in the substance. The applied force is inversely proportional to the distance between the two plates and directly proportional to the area and velocity gradient in the fluid. The above expression forms the formula below:

$$F = \mu A \frac{u}{y} \qquad 2.5$$

Where F is the Force, $\mu$ is the viscosity of the fluid, $A$ is the area of each plate and $\frac{u}{y}$ is the rate of shear deformation.

The viscosity of a material is what connects its viscous stresses to its rate of deformation change (the strain rate). Although it applies to all flows, a straightforward shearing flow, such a planar Couette flow, makes it simple to picture and define.

A fluid is caught in the Couette flow between two infinitely large plates, one of which is immobile and the other is moving parallel to it at a constant speed (See Figure 2.1).

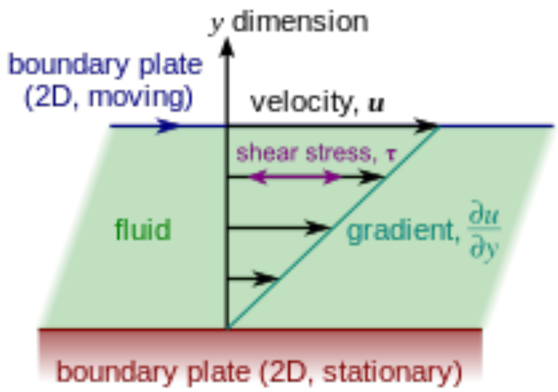


**Figure 2.1 Illustration of a planar Couette flow.**

The fluid particles flow parallel to the top plate in steady state if the top plate's speed is low enough to prevent turbulence (Mewis and Wagner, 2012). The layer of each fluid moves faster than the layer immediately below it, and friction between them creates a force that opposes their relative motion. In particular, a force is exerted on the top plate by the fluid and is opposite to the direction of motion, as well as a force that is equal to but opposite on the bottom plate.

**2.3 Classification of Flow of Fluids**

In fluid mechanics, there are five different types of fluids, namely; Ideal Fluids, Real Fluids, Newtonian Fluids, Non-Newtonian Fluids and Ideal Plastic Fluids (Dey, 2019). Due to the diversifications in the types of fluids as a result of differences in viscosity, density, velocity *et el*, no two fluids of different classifications would flow in the same way due to variations in some parameters of the fluids.

**2.3.1 Steady and Unsteady Flow.**

A fluid flow is said to be steady when its characteristics at a point do not change with time. It can be represented mathematically as;

$$\left(\frac{\partial v}{\partial t}\right) = 0, \left(\frac{\partial p}{\partial t}\right) = 0, \left(\frac{\partial J}{\partial t}\right) = 0;$$

From the expression, v represents the velocity of the fluid, p represents the pressure and J represents the density.

A fluid flow is said to be unsteady when its characteristics, unlike steady flow, changes with respect to time. It can be represented mathematically as;

$$\left(\frac{\partial v}{\partial t}\right) \neq 0, \left(\frac{\partial p}{\partial t}\right) \neq 0, \left(\frac{\partial J}{\partial t}\right) \neq 0;$$

where the v represents the velocity, p represents pressure and J represents the density of the fluid.

**2.3.2 Laminar and Turbulent Flow**

Laminar flow is the type of fluid flow in which the streamline or layer used for the movement of fluid particles is well-defined, straight and parallel. As a result, the fluid particles move in layers or laminar motion, easily gliding over one another. Small diameter pipes with high viscosity fluid and slower flow rates exhibit laminar flow. Other names for this type of flow include streamline flow and viscous flow.

Turbulent Flow is the type of fluid flow where the fluid particles travel haphazardly or in a zigzag pattern. Eddies arise as a result of the zigzag motion of fluid particles, which results in a significant loss of energy. The magnitude and direction of the fluid's speed at a given point changes continuously in turbulent flow. Turbulent flow is more likely to occur in pipes with large diameters in which the fluid tends to flow with high velocity.

### 2.3.3 Compressible and Incompressible Flow

Compressible fluid flow is the type of flow in which at any given point, the density of the fluid remains constant and Incompressible fluid flow is the type of flow in which the density of the fluid does not remain constant across several points in the fluid.

## 2.4 Definition of terms

In fluid dynamics, as there are laws that govern or fluid flow and behaviour, there are also some terms associated with the field that are also inevitable and they determine the flow characteristics of the fluid. Below is a list of terms associated with fluid mechanics that were also used during the course of this research.

### 2.4.1 Fluid velocity

It is a physical quantity that describes a fluid's motion in a mathematical manner. It is represented by the symbol, $v$.

### 2.4.2 Thermal conductivity

It is defined as the ability of the fluid to conduct heat. It is represented by the symbol, K.

### 2.4.3 Entropy generation number

Entropy generation number is the physical quantity that measures the amount of dissipated energy and the rate of degradation of systems. The rate of dissipation depends on the level of irreversibility present in the system. It is represented by the symbol, $N_S$.

### 2.4.4 Grashof number

It is a non-dimensional quantity that represents the ratio of the buoyant force (due to the spatial variation in the density of the fluid) to a viscous force acting on a fluid in the velocity boundary layer. It was named after Franz Grashof and is represented by the symbol, Gr (Hewitt *et al*, 1994).

### 2.4.5 Brinkman number

It is a non-dimensional number that's related to the heat conduction coming from a wall to a flowing viscous fluid. It is the ratio of the production of heat as a result of viscous dissipation to the movement of heat by molecular conduction. It was named after Dutch mathematician and physicist, Henri Brinkman, and is represented by the symbol, Br.

### 2.4.6 Magnetic parameter

It is a non-dimensional number that describes the ratio of viscous dissipation to thermal conduction of an MHH fluid. It is represented by the symbol, $M$.

### 2.4.7 Drag coefficient

It is a non-dimensional quantity that is used to describe the level of resistance or drag of an object in a fluid.

### 2.4.8 Volume fraction

It is a non-dimensional quantity that describes the ratio of the volume of a constituent to the volume of the whole fluid. It is represented by the symbol, $\varphi$.

### 2.4.9 Lame constant

It is second of two constants named after Gabriel Lamé that describes the dynamic viscosity of a fluid. It is represented by the symbol, $\mu$.

### 2.4.10 Viscous drag

Viscous drag is the drag force felt by an object moving through a fluid due to the viscosity of the fluid (Kaylegian-Starkey, 2022). It is represented by the symbol, $\delta$.

# CHAPTER THREE

# METHODOLOGY

## 3.0 Introduction

In this chapter, we present the model and the methodology used to derive the solution for the magnetohydrodynamic(MHD) fluid flow. Normally, the equations are in dimensional form and in order to be able to represent them, we converted them to non-dimensional form after the description of the model. Thereafter, the Adomian Decomposition Method (ADM) is applied and used to obtain the solutions of the equations.

## 3.1 The formulation of the mathematical model

Firstly, we considered a steady flow that had passed transient state, flowing through a vertical deformable porous layer bounded by two vertical plates. The $x$-axis is taken horizontally as one of the plates and the $y$-axis is chosen perpendicular to the $x$-axis. The plates are $y = h$ and $y = 0$, where $h$ is the width of the channel. The temperature at $y = 0$ will be $T = T_0$ and $T = T_w$ at $y = h$ respectively, and is maintained at constant temperatures along the flow of the fluid. The pressure of the fluid is $P_{x1}$and an outward pressure flow, $P_{x2}$ is introduced to the fluid, where the condition $P_{x2} > P_{x1}$ is necessary for the fluid to flow. Hence, the difference in pressure (pressure gradient) is $P_{x2} - P_{x1} = \frac{\partial P}{\partial x}$ which produces an axially directed flow. An external magnetic field with strength, $B_o$, is applied perpendicularly to the plates which induces electrical conductivity, $\sigma$, in the flowing field. The direction of the flow is along the positive $x$-axis with fluid velocity, $v$. The model equations are the solid displacement equation of the fluid, the velocity equation of the fluid, temperature equation of the fluid and the entropy generation equation of the fluid. The equations were obtained in dimensional form and thereafter converted to non-dimensional form. Fig 3.1 shows the physical model of the flow of the fluid:

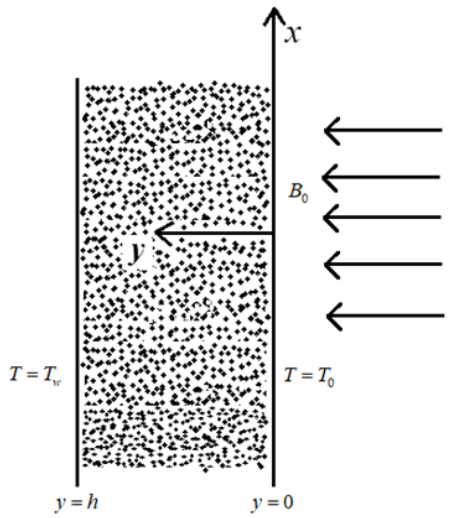


Figure 3.1: The Physical model of the flow system.

### 3.2 The model equations in dimensional form

#### 3.2.1 The solid displacement equation

$$\mu \frac{\partial^2 u^*}{\partial y^{*2}} - (1-\varphi)\frac{\partial p^*}{\partial x^*} + Kv^* = 0 \qquad 3.1$$

#### 3.2.2 The Velocity Equation

$$2\mu_a \frac{\partial^2 v^*}{\partial y^{*2}} - \varphi \frac{\partial p^*}{\partial x^*} - Kv^* - \sigma B_o v^* + g\rho\beta(T - T_0) = 0 \qquad 3.2$$

#### 3.2.3 The temperature equation

$$K_o \frac{\partial^2 T}{\partial y^{*2}} + Q_o = 0 \qquad 3.3$$

#### 3.2.4 The entropy generation equation

$$E_G = \frac{K_o}{T_0^2}\left(\frac{dT}{dy^*}\right)^2 + \frac{\mu_f}{T_0}\left(\frac{dv^*}{dy^*}\right)^2 + \frac{\sigma B_o^2}{T_0} v^{*2} \qquad 3.4$$

Where ; $\mu$ is the lame constant, K is the drag coefficient, $\varphi$ is the volume fraction of the fluid, $v^*$ is the dimensional form of the velocity of the fluid, $\mu_a$ is the apparent viscosity of fluid in the porous material, $\sigma$ is the electrical conductivity of the fluid, $g$ is the acceleration due to gravity of the fluid, $T$ is the temperature of the fluid, $K_o$is the thermal conductivity of the fluid, $Q_o$ is the heat source, $\mu_f$ is the coefficient of viscosity, and $x^*$ and $y^*$ are the dimensional form of the Cartesian coordinates.

### 3.3 The non-dimensional form of the equations

The non-dimensional quantities;

$$y = \frac{y^*}{h}, x = \frac{x^*}{h}, v = \frac{v^*}{U}, \theta = \frac{T-T_0}{T_w-T_0}, p = \frac{hp^*}{\mu_f U}, u = \frac{u^*\mu}{\mu_f U}, \mu_f = 2\mu_a, \delta = \frac{Kh^2}{\mu_f},$$

$$Gr = \frac{g\beta(T_w-T_0)\rho h^2}{\mu_f U}, \eta = \frac{\mu_f}{2\mu_a}, \beta = \frac{Q_o h^2}{K_o(T-T_0)}, M = \sqrt{\frac{h^2\sigma B_o^2}{\mu_f \rho}}$$

#### 3.3.1 The non-dimensional solid displacement equation

From equation 3.1,

$$\mu\frac{\partial^2 u^*}{\partial y^{*2}} - (1-\varphi)\frac{\partial p^*}{\partial x^*} + Kv^* = 0$$

From the non-dimensional quantity relation:$y = \frac{y^*}{h}$;

$$\frac{d}{dy^*} = \frac{1}{h}\frac{d}{dy}; \frac{d^2}{dy^{*2}} = \frac{1}{h^2}\frac{d^2}{dy^2} \quad 3.5$$

$$x = \frac{x^*}{h}; x^* = xh, \quad 3.6$$

$$v = \frac{v^*}{U}; v^* = vU, \quad 3.7$$

$$u = \frac{u^*\mu}{\mu_f U}; u^* = \frac{u\mu_f U}{\mu} \quad 3.8$$

Putting equations 3.5, 3.6, 3.7 and 3.8 into equation 3.1;

$$\mu\left(\frac{U\mu_f}{\mu h^2}\right)\frac{d^2u}{dy^2} - (1-\varphi)\frac{\partial p^*}{\partial xh} + KvU = 0 \quad 3.9$$

Multiplying through equation 3.9 by $\frac{h^2}{U\mu_f}$

$$\frac{d^2u}{dy^2} - (1-\varphi)\frac{\partial p^*}{\partial xh}\frac{h^2}{U\mu_f} + \frac{KvUh^2}{U\mu_f} = 0 \quad 3.10$$

$$\frac{d^2u}{dy^2} - (1-\varphi)\frac{d}{dx^*}\left(\frac{p^*h^2}{U\mu_f}\right) + \frac{h^2Kv}{\mu_f} = 0 \quad 3.11$$

$$x = \frac{x^*}{h}; x^* = xh$$

Applying the above expression into equation 3.11

$$\frac{d^2u}{dy^2} - (1-\varphi)\frac{d}{dxh}\left(\frac{p^*h^2}{U\mu_f}\right) + \frac{h^2Kv}{\mu_f} = 0 \quad 3.12$$

$$\frac{d^2u}{dy^2} - (1-\varphi)\frac{d}{dx}\left(\frac{p^*h}{U\mu_f}\right) + \frac{h^2Kv}{\mu_f} = 0 \quad 3.13$$

Recall $p = \frac{hp^*}{U\mu_f}$, $\delta = \frac{h^2K}{\mu_f}$, and applying it into equation 3.13

$$\frac{d^2u}{dy^2} - (1-\varphi)\frac{dp}{dx} + \delta v = 0 \quad 3.14$$

$$\frac{d^2u}{dy^2} - (1-\varphi)P + \delta v = 0. \quad 3.15$$

Equation 3.15 is the expression of the displacement of the MHD fluid in the deformable porous material, where $P = \frac{dp}{dx}$

### 3.3.2 The non-dimensional velocity equation

From equation 3.2;

$$2\mu_a \frac{\partial^2 v^*}{\partial y^{*2}} - \varphi \frac{\partial p^*}{\partial x^*} - Kv^* - \sigma B_o^2 v^* + g\rho\beta(T - T_0) = 0$$

From the non-dimensional quantity relations: $\frac{\partial}{\partial y^2} = \frac{1}{h^2}\frac{d}{dy^2}$ , $v = \frac{v^*}{U}$ ; $v^* = vU$ , $\theta = \frac{T - T_0}{T_w - T_0}$ ;

$T - T_0 = \theta(T_w - T_0)$ and applying it to equation 3.2, we have

$$\frac{2\mu_a U}{h^2}\frac{d^2 v}{dy^2} - \varphi \frac{\partial p^*}{\partial x^*} - KvU - \sigma B_o^2 vU + g\rho\beta\theta(T_w - T_0) = 0 \qquad 3.16$$

Multiplying equation 3.16 through by $\frac{h^2}{2\mu_a U}$

$$\frac{d^2 v}{dy^2} - \varphi \frac{h^2}{2\mu_a U}\frac{\partial p^*}{\partial x^*} - \frac{h^2 KvU}{2\mu_a U} - \frac{h^2 \sigma B_o^2 vU}{2\mu_a U} + \frac{h^2 g\rho\beta\theta(T_w - T_0)}{2\mu_a U} = 0 \qquad 3.17$$

$$\frac{d^2 v}{dy^2} - \varphi \frac{h^2}{\mu_f U}\frac{\partial p^*}{\partial x^*} - \frac{h^2 K}{\mu_f} v - \frac{h^2 \sigma B_o^2 v}{\mu_f} + \frac{h^2 g\rho\beta(T_w - T_0)}{2\mu_a U}\theta = 0 \qquad 3.18$$

From the non-dimensional quantity relations: $\delta = \frac{h^2 K}{\mu_f}, Gr = \frac{h^2 g\rho\beta(T_w - T_0)}{2\mu_a U}$

$$\frac{d^2 v}{dy^2} - \varphi \frac{h^2}{\mu_f U}\frac{\partial p^*}{\partial x^*} - \delta v - \frac{h^2 \sigma B_o^2 v}{\mu_f} + Gr\theta = 0 \qquad 3.19$$

$$\frac{d^2 v}{dy^2} - \varphi \frac{\partial}{\partial x^*}\left(\frac{p^* h^2}{\mu_f U}\right) - \delta v - \frac{h^2 \sigma B_o^2 v}{\mu_f} + Gr\theta = 0 \qquad 3.20$$

Applying the following non-dimensional relations to equation 3.20: $x^* = xh, p^* = \frac{hp}{\mu_f U}$

$$\frac{d^2 v}{dy^2} - \varphi \frac{\partial p}{\partial x} - \delta v - \frac{h^2 \sigma B_o^2}{\mu_f} v + Gr\theta = 0 \qquad 3.21$$

$$M^2 = \frac{h^2 \sigma B_o^2 v}{\mu_f}; M = \sqrt{\frac{h^2 \sigma B_o^2}{\mu_f}} , P = \frac{\partial p}{\partial x}$$

$$\frac{d^2 v}{dy^2} - \varphi P - \delta v - Mv + Gr\theta = 0$$

$$\frac{d^2 v}{dy^2} - \varphi P\eta - (\delta + M)\eta v + Gr\eta\theta = 0 \qquad 3.22$$

Equation 3.22 is the non-dimensional expression of the velocity of the MHD fluid in the porous material, where $\eta = \frac{\mu_f}{2\mu_a} = 1$ and is present in the equation to show the ratio of bulk fluid viscosity to apparent fluid viscosity in porous layer.

### 3.3.3 The non-dimensional temperature equation

From equation 3.3;

$$K_o \frac{\partial^2 T}{\partial y^{*2}} + Q_o = 0$$

Dividing equation 3.3 through by $K_o$;

$$\frac{\partial^2 T}{\partial y^{*2}} + \frac{Q_o}{K_o} = 0 \qquad 3.23$$

Recall the non-dimensional quantity: $\frac{\partial}{\partial y^2} = \frac{1}{h^2}\frac{d}{dy^2}$

$$\frac{1}{h^2}\frac{d^2 T}{dy^2} + \frac{Q_o}{K_o} = 0 \qquad 3.24$$

$$\frac{d^2 T}{dy^2} + \frac{h^2 Q_o}{K_o} = 0 \qquad 3.25$$

From the non-dimensional quantity: $\theta = \frac{T - T_0}{T_w - T_0}$;

$$T - T_0 = \theta(T_w - T_0) \qquad 3.26$$

$$T = \theta(T_w - T_0) + T_0 \qquad 3.27$$

Applying equation 3.27 into equation 3.25

$$\frac{d^2(\theta(T_w - T_0) + T_0)}{dy^2} + \frac{h^2 Q_o}{K_o} = 0 \qquad 3.28$$

$$\frac{d^2}{dy^2}(\theta(T_w - T_0) + T_0) + \frac{h^2 Q_o}{K_o} = 0 \qquad 3.29$$

$Leta = T_w - T_0, b = T_0$

$a\theta - b = \theta(T_w - T_0) + T_0$

Applying the above expression into equation 3.29

$$\frac{d^2}{dy^2}(a\theta - b) + \frac{h^2 Q_o}{K_o} = 0 \qquad 3.30$$

$$\frac{d^2}{dy^2}(a\theta) - \frac{d^2}{dy^2}(b) + \frac{h^2 Q_o}{K_o} = 0$$

$$\frac{d^2}{dy^2}(a\theta) - \frac{d^2}{dy^2}0 + \frac{h^2 Q_o}{K_o} = 0$$

$$\frac{d^2}{dy^2}(a\theta) + \frac{h^2 Q_o}{K_o} = 0$$

$$\frac{ad^2\theta}{dy^2} + \frac{h^2 Q_o}{K_o} = 0$$

$$\frac{(T_w - T_0)d^2\theta}{dy^2} + \frac{h^2 Q_o}{K_o} = 0 \text{ , where } a = T_w - T_0 \qquad 3.31$$

Dividing equation 3.31 through by $(T_w - T_0)$

$$\frac{d^2\theta}{dy^2} + \frac{h^2 Q_o}{K_o(T_w - T_0)} = 0 \qquad 3.32$$

$$Let\alpha = \frac{h^2 Q_o}{K_o(T_w - T_0)}$$

$$\frac{d^2\theta}{dy^2} + \alpha = 0 \qquad 3.33$$

Equation 3.33 above is the equation for the temperature of the MHD fluid in the deformable porous material, where $\alpha$ is the heat source and is the non-dimensional expression of temperature.

### 3.3.4 The non-dimensional entropy equation

From equation 3.4 above;

$$E_G = \frac{K_o}{T_0^2}\left(\frac{dT}{dy^*}\right)^2 + \frac{\mu_f}{T_0}\left(\frac{dv^*}{dy^*}\right)^2 + \frac{\sigma B_o^2}{T_0} v^{*2}$$

Multiplying equation 3.4 through by $T_0^2$

$$T_0^2 E_G = K_o\left(\frac{dT}{dy^*}\right)^2 + T_0\mu_f\left(\frac{dv^*}{dy^*}\right)^2 + \sigma B_o^2 v^{*2} T_0 \qquad 3.34$$

Applying the following non-dimensional quantity relations to equation 3.34;

$\frac{d}{dy} = \frac{1}{h}\frac{d}{dy}$ , $\theta = \frac{T-T_0}{T_w-T_0}$ ; $T - T_0 = \theta(T_w - T_0)\,; T = \theta(T_w - T_0) + T_0\,; T_0 = \theta(T_w - T_0) + T$ ,

$v = \frac{v^*}{U}\,; v^* = vU, x = \frac{x^*}{h}\,; y^* = yh$

$$T_0^2 E_G = K_o\left(\frac{1}{h}\frac{d(\theta(T_w-T_0)+T_0)}{dy}\right)^2 + T_0\mu_f\left(\frac{1}{h}\frac{dvU}{dy}\right)^2 + \sigma B_o^2 v^2 U^2 T_0 \qquad 3.35$$

$$T_0^2 E_G = \frac{K_o}{h^2}\left((T_w - T_0)\frac{d\theta}{dy}\right)^2 + \frac{T_0\mu_f U^2}{h^2}\left(\frac{dv}{dy}\right)^2 + \sigma B_o^2 v^2 U^2 T_0$$

$$T_0^2 E_G = \frac{K_o(T_w-T_0)^2}{h^2}\left(\frac{d\theta}{dy}\right)^2 + \frac{T_0\mu_f U^2}{h^2}\left(\frac{dv}{dy}\right)^2 + \sigma B_o^2 v^2 U^2 T_0 \qquad 3.36$$

Multiplying equation 3.36 through by $\frac{h^2}{K_o(T_w-T_0)^2}$

$$\frac{T_0^2 E_G h^2}{K_o(T_w-T_0)^2} = \left(\frac{d\theta}{dy}\right)^2 + \frac{h^2 T_0\mu_f U^2}{h^2 K_o(T_w-T_0)^2}\left(\frac{dv}{dy}\right)^2 + \frac{h^2\sigma B_o^2 v^2 U^2 T_0}{K_o(T_w-T_0)^2}$$

Let entropy generation, $N_s = \frac{T_0^2 E_G h^2}{K_o(T_w-T_0)^2}$;

$$N_s = \left(\frac{d\theta}{dy}\right)^2 + \frac{T_0\mu_f U^2}{K_o(T_w-T_0)^2}\left(\frac{dv}{dy}\right)^2 + \frac{h^2\sigma B_o^2 v^2 U^2 T_0}{K_o(T_w-T_0)^2} \qquad 3.37$$

$$N_s = \left(\frac{d\theta}{dy}\right)^2 + \frac{T_0\mu_f U^2}{K_o(T_w-T_0)^2}\left(\frac{dv}{dy}\right)^2 + \frac{U^2 T_0\mu_f}{K_o(T_w-T_0)^2}\frac{h^2\sigma B_o^2}{\mu_f}v^2 \qquad 3.38$$

Recall; $M^2 = \frac{h^2\sigma B_o^2 v}{\mu_f}; M = \sqrt{\frac{h^2\sigma B_o^2}{\mu_f}}$ and applying it into equation 3.38

$$N_s = \left(\frac{d\theta}{dy}\right)^2 + \frac{T_0\mu_f U^2}{K_o(T_w-T_0)^2}\left(\frac{dv}{dy}\right)^2 + \frac{U^2 T_0\mu_f}{K_o(T_w-T_0)^2}Mv^2 \qquad 3.39$$

$$N_s = \left(\frac{d\theta}{dy}\right)^2 + \frac{U^2 T_0\mu_f}{K_o(T_w-T_0)^2}\left(\left(\frac{dv}{dy}\right)^2 + Mv^2\right)$$

Let $Br = \frac{U^2\mu_f}{K_o(T_w-T_0)}$ , $\Omega = \frac{(T_w-T_0)}{T_0}; \frac{1}{\Omega} = \frac{T_0}{(T_w-T_0)}$

$$N_s = \left(\frac{d\theta}{dy}\right)^2 + \frac{Br}{\Omega}\left(\left(\frac{dv}{dy}\right)^2 + Mv^2\right) \qquad 3.40$$

Equation 3.40 is the non-dimensionless equation for the entropy generation of the MHD fluid flowing in the deformable porous material, where Br is the Brinkman number and $\Omega$ is the non-dimensional temperature difference.

### 3.4 Adomian decomposition method

In this section, the four non-dimensional equations (equation 3.15, 3.22, 3.33 and 3.40) were cast into a recursive scheme using the ADM in order to solve for the solid displacement, velocity, temperature and entropy generation number.

The following boundary conditions:

$$\text{At } y = 0; u = 0, v = 0, \theta = 0$$

$$\text{At } y = 1; u = 0, v = 0, \theta = 1$$

were used to solve the non-dimensional equations using ADM.

#### 3.4.1 Recursive solution for solid displacement

From equation 3.15, $\frac{d^2u}{dy^2} - (1 - \varphi)P + \delta v = 0$, and making the term with the highest order of differential coefficient the subject of the formula;

$$\frac{d^2u}{dy^2} = (1 - \varphi)P - \delta v \qquad 3.41$$

Let $L_y \equiv \frac{d^2}{d^2y}$ , where $L_y$ is an operator that's a function of y.

∴ the inverse operator would be $L_y^{-1} = \int_0^y \int_0^y (*)dydy$, where $(*) = \frac{d^2u}{d^2y}$

Equation 3.41 then becomes:

$$L_y u = (1 - \varphi)P - \delta v \qquad 3.42$$

Applying the inverse operator to both sides of equation 3.42

$L_y^{-1}(L_y u) = L_y^{-1}[(1-\varphi)P - \delta v]$ 3.43

$L_y u = ?$

Recall : $(*) = \frac{d^2u}{d^2y}$ 3.44

Integrating both sides of equation 3.44;

$\int_0^y \frac{d^2u}{d^2y} dy = \int_0^y (*) dy$

$\frac{du}{dy} = \frac{du}{dy}\Big|_{y=0} + \int_0^y (*) dy$ 3.45

Integrating both sides of equation 3.45;

$$u = u|_{y=0} + \int_0^y \frac{du}{dy}\Big|_{y=0} dy + \int_0^y \int_0^y (*) dy\, dy$$

$u = u(0) + \int_0^y \frac{du(0)}{dy} dy + \int_0^y \int_0^y (*) dy\, dy$ 3.46

Applying the boundary conditions, $u = 0$ when $y = 0$, to equation 3.46

$u = \int_0^y \frac{du(0)}{dy} dy + \int_0^y \int_0^y (*) dy\, dy$

Let $\frac{du(0)}{dy} = f$

$u = \int_0^y f\, dy + \int_0^y \int_0^y (*) dy\, dy$

$u = fy + L_y^{-1} L_y u$ 3.47

$L_y^{-1} L_y u = u - fy$ 3.48

Putting equation 3.48 into equation 3.43

$u - fy = L_y^{-1}[(1-\varphi)P - \delta v]$

$u = fy + L_y^{-1}(1-\varphi)P - L_y^{-1}\delta v$ 3.49

$u = fy + \int_0^y \int_0^y (1-\varphi)P dy dy - L_y^{-1}\delta v$ , where $L_y^{-1}(1-\varphi)P = \int_0^y \int_0^y (1-\varphi)P dy dy$

$$u = fy + \int_0^y (1-\varphi)Pydy - L_y^{-1}\delta v$$

$$u = fy + \frac{1}{2}(1-\varphi)Py^2 - L_y^{-1}\delta v$$

$$u = fy + \frac{1}{2}(1-\varphi)Py^2 - \delta L_y^{-1} v \qquad 3.50$$

Let $u = u_0 + u_1 + u_2 + u_3 \ldots$

$$u_0 + u_1 + u_2 + \ldots ky + \frac{1}{2}(1-\varphi)Py^2 - \delta L_y^{-1} v \qquad 3.51$$

$u_0 = fy + \frac{1}{2}(1-\varphi)Py^2$ because the values of $u_0$ are dependent on y.

Let $u_{k+1} = -\delta L_y^{-1} v_k; k \geq 0$

$u_1 = -\delta L_y^{-1} v_0$

$u_2 = -\delta L_y^{-1} v_1$

$u_3 = -\delta L_y^{-1} v_2$

For ADM, $u = \sum_{n=0}^{\infty} u_n$.

Hence, the recursive solution for the non-dimensional equation for the solid displacement of the fluid would be;

$u(y) = u_0 + u_1 + u_2 + u_3 \ldots + u_\infty$

**3.4.2 Recursive solution for velocity**

From equation 3.22, $\frac{d^2v}{dy^2} - \varphi P\eta - (\delta + M)\eta v + Gr\eta\theta = 0$, and making the term with the highest order of differential coefficient the subject of the formula;

$$\frac{d^2v}{dy^2} = \varphi P\eta + (\delta + M)\eta v - Gr\eta\theta \qquad 3.52$$

Let $L_y \equiv \frac{d^2}{d^2y}$ , where $L_y$ is an operator that's a function of y.

$\therefore$ the inverse operator would be $L_y^{-1} = \int_0^y \int_0^y (*) dy dy$, where $(*) = \frac{d^2v}{d^2y}$

Equation 3.52 then becomes:

$$L_y v = \varphi P\eta + (\delta + M)\eta v - Gr\eta\theta \qquad 3.53$$

Applying the inverse operator to both sides of equation 3.53

$$L_y^{-1}(L_y u) = L_y^{-1}[\varphi P\eta + (\delta + M)\eta v - Gr\eta\theta] \qquad 3.54$$

$L_y v = ?$

Recall : $(*) = \frac{d^2v}{d^2y}$ 3.55

Integrating both sides of equation 3.55;

$$\int_0^y \frac{d^2v}{d^2y} dy = \int_0^y (*) dy$$

$$\frac{dv}{dy} = \left.\frac{dv}{dy}\right|_{y=0} + \int_0^y (*) dy \qquad 3.56$$

Integrating both sides of equation 3.56;

$$v = v|_{y=0} + \int_0^y \left.\frac{dv}{dy}\right|_{y=0} dy + \int_0^y \int_0^y (*) dy\, dy$$

$$v = v(0) + \int_0^y \frac{dv(0)}{dy} dy + \int_0^y \int_0^y (*) dy\, dy \qquad 3.57$$

Applying the boundary conditions, $v = 0$ when $y = 0$, to equation 3.57

$$v = \int_0^y \frac{dv(0)}{dy} dy + \int_0^y \int_0^y (*) dy\, dy$$

Let $\frac{dv(0)}{dy} = j$

$$v = \int_0^y j\, dy + \int_0^y \int_0^y (*) dy\, dy$$

$$v = jy + L_y^{-1} L_y v$$

$$L_y^{-1}L_y u = v - jy \quad 3.58$$

Putting equation 3.58 into 3.54;

$$v - jy = L_y^{-1}[\varphi P\eta + (\delta + M)\eta v - Gr\eta\theta]$$

$$v = L_y^{-1}[\varphi P\eta + (\delta + M)\eta v - Gr\eta\theta] + jy$$

$$v = L_y^{-1}\varphi P\eta + (\delta + M)\eta L_y^{-1}v - Gr\eta L_y^{-1}\theta + jy$$

$$v = L_y^{-1}\varphi P\eta + (\delta + M)\eta L_y^{-1}v - Gr\eta L_y^{-1}\theta + jy$$

$$v = \int_0^y \int_0^y \varphi P\eta dydy + (\delta + M)\eta L_y^{-1}v - Gr\eta L_y^{-1}\theta + jy$$

where $L_y^{-1}\varphi P\eta = \int_0^y \int_0^y \varphi P\eta dydy$

$$v = \int_0^y \varphi P\eta ydy + (\delta + M)\eta L_y^{-1}v - Gr\eta L_y^{-1}\theta + jy$$

$$v = \frac{1}{2}\varphi P\eta y^2 + (\delta + M)\eta L_y^{-1}v - Gr\eta L_y^{-1}\theta + jy \quad 3.59$$

Let $v = v_0 + v_1 + v_2 + v_3 \ldots$

$$v_0 + v_1 + v_2 + v_3 \ldots = \frac{1}{2}\varphi P\eta y^2 + (\delta + M)\eta L_y^{-1}v - Gr\eta L_y^{-1}\theta + jy \quad 3.60$$

$v_0 = \frac{1}{2}\varphi P\eta y^2 + jy$ because the values of $v_0$ are dependent on y.

Let $v_{k+1} = (\delta + M)\eta L_y^{-1}v_k - Gr\eta L_y^{-1}\theta_k; k \geq 0$

$$v_1 = (\delta + M)\eta L_y^{-1}v_0 - Gr\eta L_y^{-1}\theta_0$$

$$v_2 = (\delta + M)\eta L_y^{-1}v_1 - Gr\eta L_y^{-1}\theta_1$$

$$v_3 = (\delta + M)\eta L_y^{-1}v_2 - Gr\eta L_y^{-1}\theta_2$$

For ADM, $v = \sum_{n=0}^{\infty} v_n$

Hence, the recursive solution for the non-dimensional equation for the solid displacement of the fluid would be; $v(y) = v_0 + v_1 + v_2 + v_3 \ldots + v_\infty$

### 3.4.3 Recursive solution for temperature

From equation 3.33, $\frac{d^2\theta}{dy^2} + \alpha = 0$, and making the term with the highest order of differential coefficient the subject of the formula;

$$\frac{d^2\theta}{dy^2} = -\alpha \quad 3.61$$

Let $L_y \equiv \frac{d^2}{d^2y}$ , where $L_y$ is an operator that's a function of y.

$\therefore$ the inverse operator would be $L_y^{-1} = \int_0^y \int_0^y (*) dydy$, where $(*) = \frac{d^2\theta}{d^2y}$

Equation 3.61 then becomes:

$$L_y\theta = -\alpha \quad 3.62$$

Applying the inverse operator to both sides of equation 3.62

$$L_y^{-1}(L_y\theta) = L_y^{-1}[-\alpha] \quad 3.63$$

$L_y\theta =?$

$$\text{Recall : } (*) = \frac{d^2\theta}{d^2y} \quad 3.64$$

Integrating both sides of equation 3.64;

$$\int_0^y \frac{d^2\theta}{d^2y} dy = \int_0^y (*) dy$$

$$\frac{d\theta}{dy} = \left.\frac{d\theta}{dy}\right|_{y=0} + \int_0^y (*) dy \quad 3.65$$

Integrating both sides of equation 3.65;

$$\theta = \theta|_{y=0} + \int_0^y \left.\frac{d\theta}{dy}\right|_{y=0} dy + \int_0^y \int_0^y (*) dy\, dy$$

$\theta = \theta(0) + \int_0^y \frac{d\theta(0)}{dy} dy + \int_0^y \int_0^y (*) dy\, dy$ 3.66

Applying the boundary conditions, $\theta = 0$ when $y = 0$, to equation 3.66

$\theta = \int_0^y \frac{d\theta(0)}{dy} dy + \int_0^y \int_0^y (*) dy\, dy$

Let $\frac{d\theta(0)}{dy} = q$

$\theta = \int_0^y q\, dy + \int_0^y \int_0^y (*) dy\, dy$

$\theta = qy + L_y^{-1} L_y \theta$

$L_y^{-1} L_y \theta = \theta - qy$ 3.67

Putting equation 3.67 into 3.63

$\theta - qy = L_y^{-1}[-\alpha]$ 3.68

$\theta = L_y^{-1}[-\alpha] + qy$

$$\theta = \int_0^y \int_0^y (-\alpha) dy\, dy + qy$$

$$\theta = \int_0^y (-\alpha) y dy + qy$$

$\theta = \frac{-1}{2} \alpha y^2 + qy$ 3.69

Let $\theta = \theta_0 + \theta_1 + \theta_2 + \theta_3 \ldots$

$\theta_0 + \theta_1 + \theta_2 + \theta_3 = \frac{-1}{2} - \alpha y^2 + qy$ 3.70

$\theta_0 = \frac{-1}{2} \alpha y^2 + qy$ because the values of $\theta_0$ are dependent on y,

The values for $\theta_1$, $\theta_2$ and $\theta_3$ would be zero since there are no other variables in the equation that's not dependent on y.

For ADM, $\theta = \sum_{n=0}^{\infty} \theta_n$

Hence, the recursive solution for the non-dimensional equation for the solid displacement of the fluid would be; $\theta(y) = \theta_0 + \theta_1 + \theta_2 + \theta_3 \ldots + \theta_\infty$ ,which would be; $\theta = \frac{-1}{2} \alpha y^2 + qy$

# CHAPTER FOUR

# RESULTS AND DISCUSSION

## 4.0 Introduction

The magnetohydrodynamic(MHD) fluid flowing in the deformable porous material was subjected to several instances and were then analysed for a number of control parameters. The values of parameters such as the magnetic parameter (M), heat source ($\alpha$), ratio of bulk fluid viscosity to apparent fluid viscosity in porous layer ($\eta$), viscous drag($\delta$), Grashof number(Gr), volume fraction of the fluid($\varphi$), varied and their effects on the velocity, solid displacement, temperature and entropy generation number of the fluid were solved and represented graphically with the aid of MATHEMATICA, a mathematical computation program used in many scientific, engineering, mathematical, and computing fields.

## 4.1 Analysis of the Effects of the Control Parameters on the Fluid

Using MATHEMATICA, a number of calculations were made to see how it would affect some physical variables of the fluid such as the solid displacement, velocity, temperature and entropy. The results of the calculations formed the basis of the argument that an MHD fluid flowing steadily in a deformable porous media will differ under different conditions.

### 4.1.1 Analysis of the Effects of the Control Parameters on the Solid Displacement

The following graphs represent effects of the varying values of different parameters: ratio of the bulk fluid velocity on the solid displacement ($\eta$), heat source ($\alpha$), magnetic parameter (M), volume fraction of the fluid($\varphi$), viscous drag($\delta$), and Grashof number(Gr), on the solid displacement of the fluid.

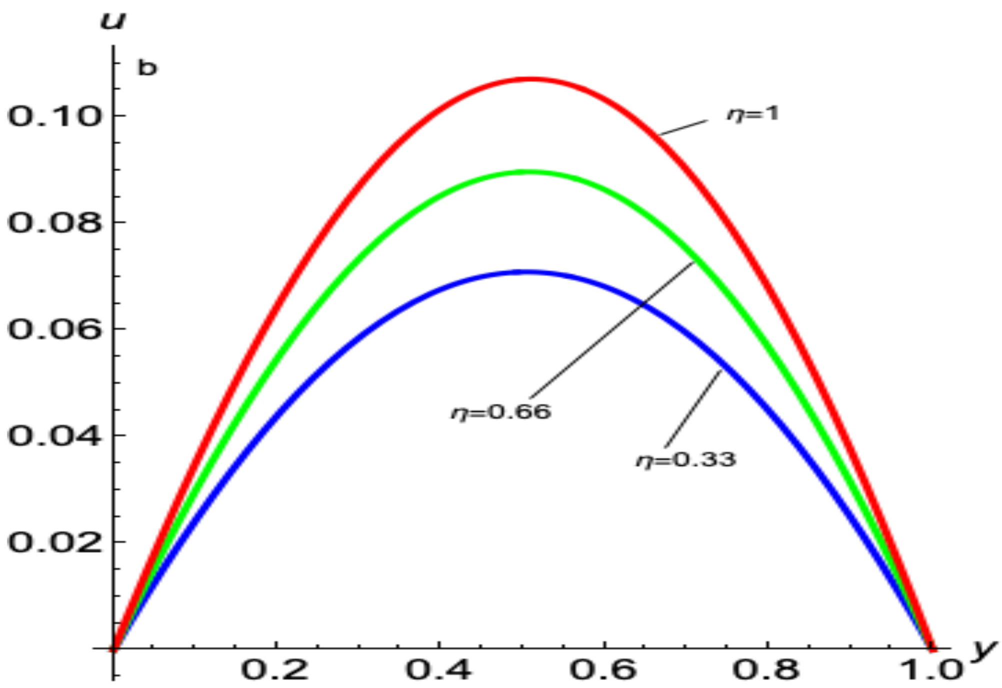


**Figure 4.1 A graph of displacement(u) against width(y) for varying values of ratio of the bulk fluid velocity on the solid displacement ($\eta$).**

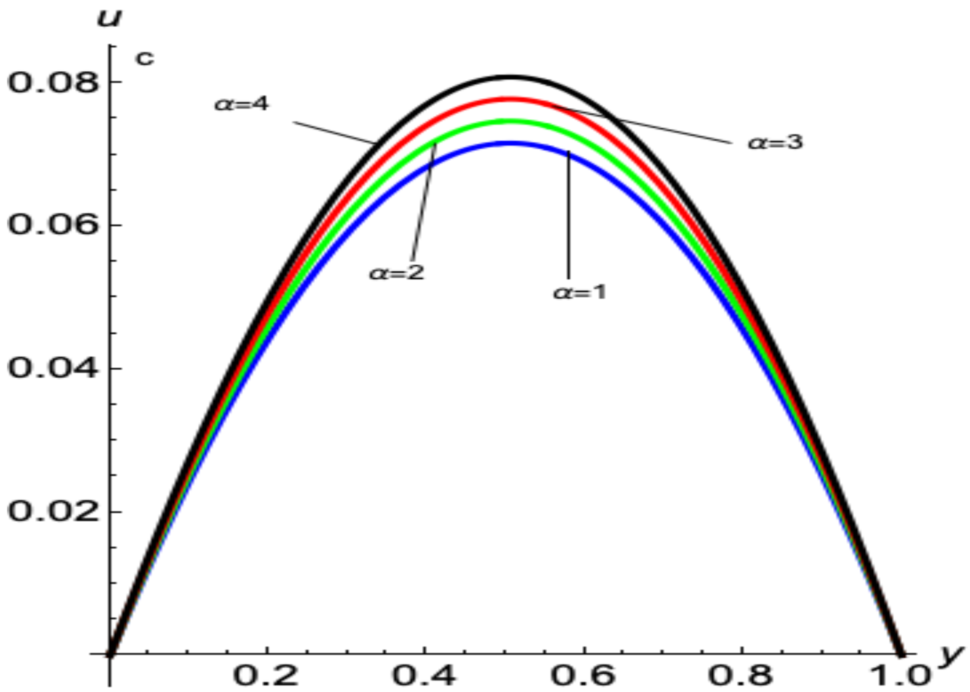


**Figure 4.2. A graph of displacement(u) against width(y) for varying values of heat source ($\alpha$).**

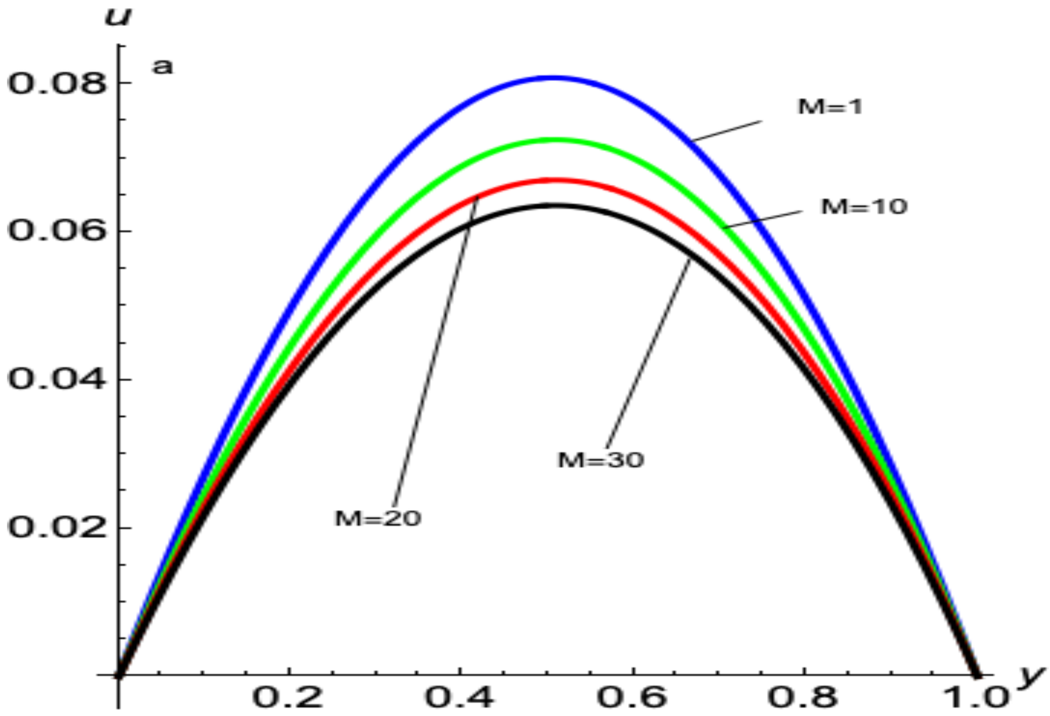


**Figure 4.3. A graph of displacement(u) against width(y) for varying values of magnetic parameter (M).**

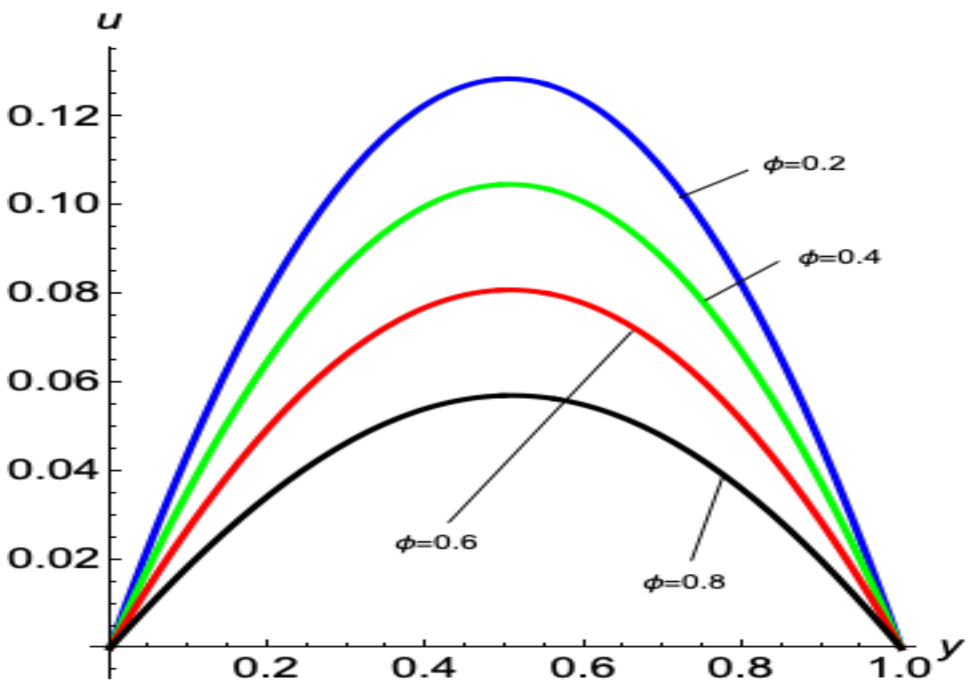


**Figure 4.4. A graph of displacement(u) against width(y) for varying values of volume fraction of the fluid($\varphi$).**

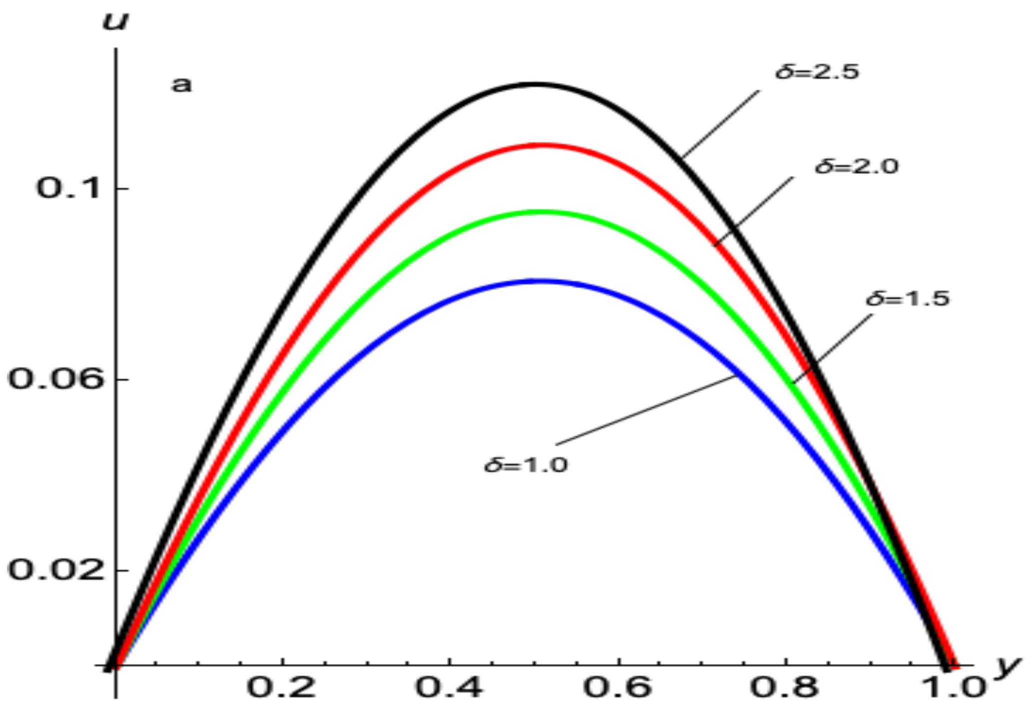


**Figure 4.5. A graph of displacement(u) against width(y) for varying values of viscous drag($\delta$).**

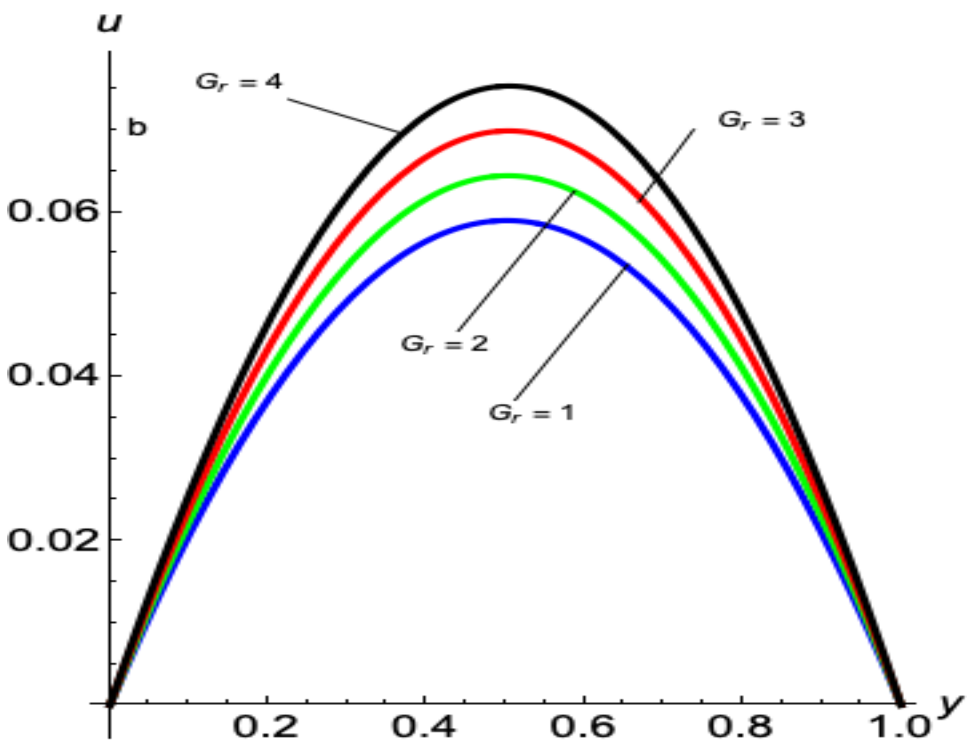


**Figure 4.6. A graph of displacement(u) against width(y) for varying values of Grashof number(Gr).**

Figures 4.1, 4.2, 4.3, 4.4, 4.5 and 4.6 are plots of displacement of the fluid against the width of the material with varying values of ratio of the bulk fluid velocity on the solid displacement ($\eta$), heat source ($\alpha$), magnetic parameter (M), volume fraction of the fluid($\varphi$), viscous drag($\delta$), and Grashof number(Gr) of the fluid respectively. For every increase in ratio of the bulk fluid velocity on the solid displacement, the displacement of the fluid increases. Also, for every increase in the heat source ($\alpha$), the displacement of the fluid increases. It was also observed that an increment in the magnetic parameter(M) and volume fraction($\varphi$) of the fluid displacement of the fluid resulted in the decrement of the displacement of the fluid. Also, the displacement of the fluid increased with an increase in the values of the viscous drag($\delta$) of the fluid and Grashof number(Gr).

### 4.1.2 Analysis of the Effects of the Control Parameters on the Velocity

The following graphs represent effects of the varying values of different parameters: ratio of the bulk fluid velocity on the solid displacement ($\eta$), heat source ($\alpha$), magnetic parameter (M), volume fraction of the fluid($\varphi$), viscous drag($\delta$), and Grashof number(Gr), on the velocity of the fluid.

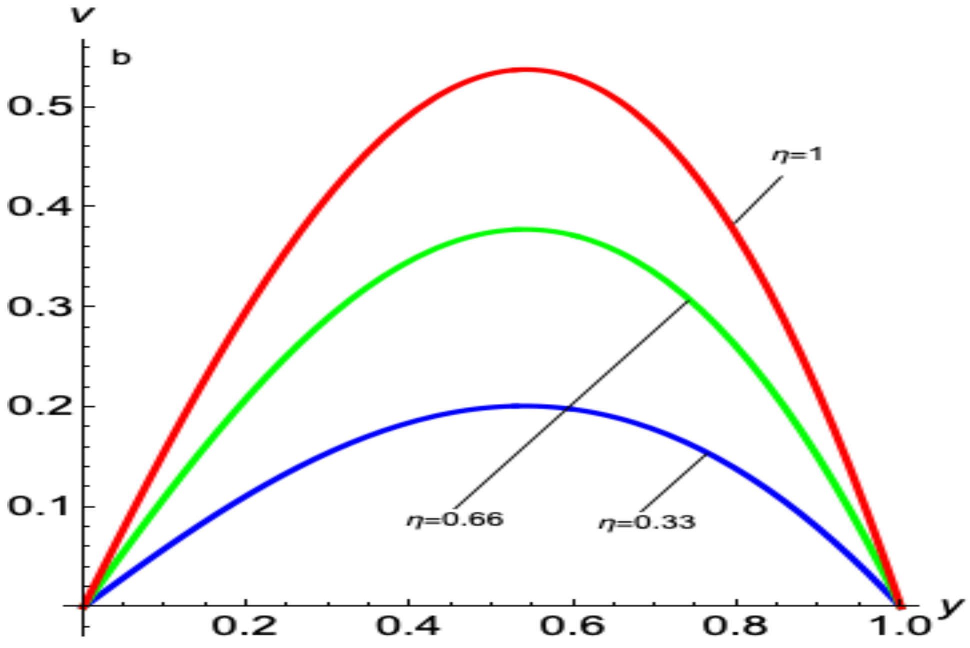


**Figure 4.7. A graph of velocity(v) against width(y) for varying values of ratio of the bulk fluid velocity on the solid displacement ($\eta$).**

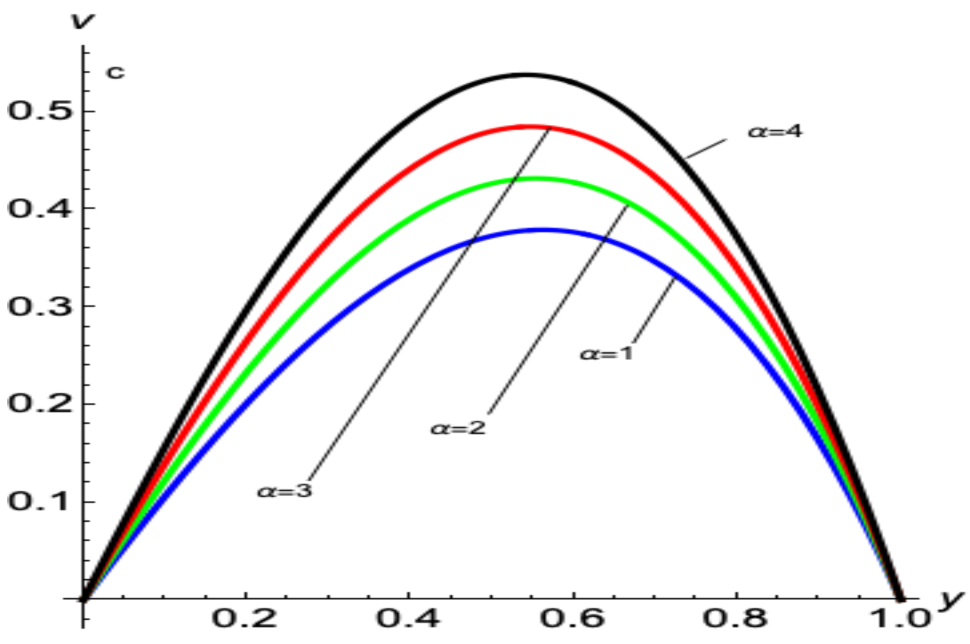


**Figure 4.8. A graph of velocity(v) against width(y) for varying values of heat source ($\alpha$).**

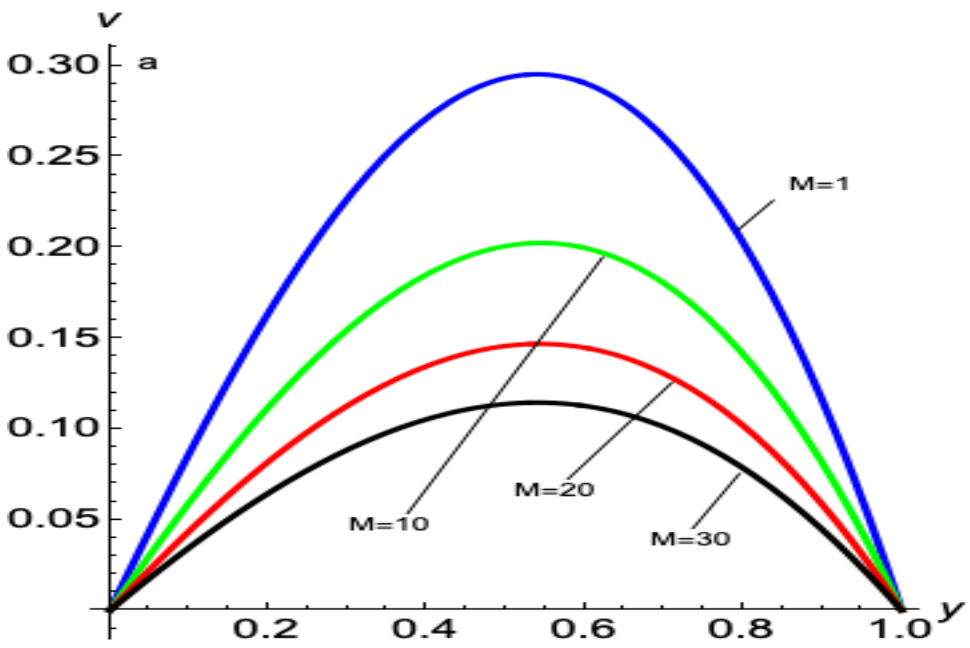


**Figure 4.9. A graph of velocity(v) against width(y) for varying values of magnetic parameter (M).**

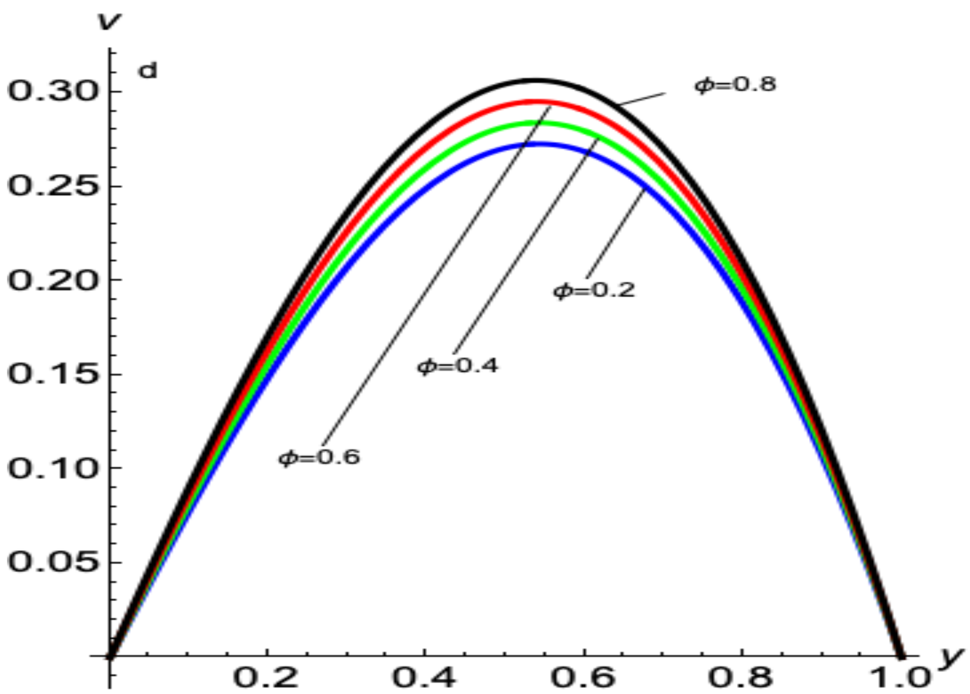


**Figure 4.10. A graph of velocity(v) against width(y) for varying values of volume fraction of the fluid($\varphi$).**

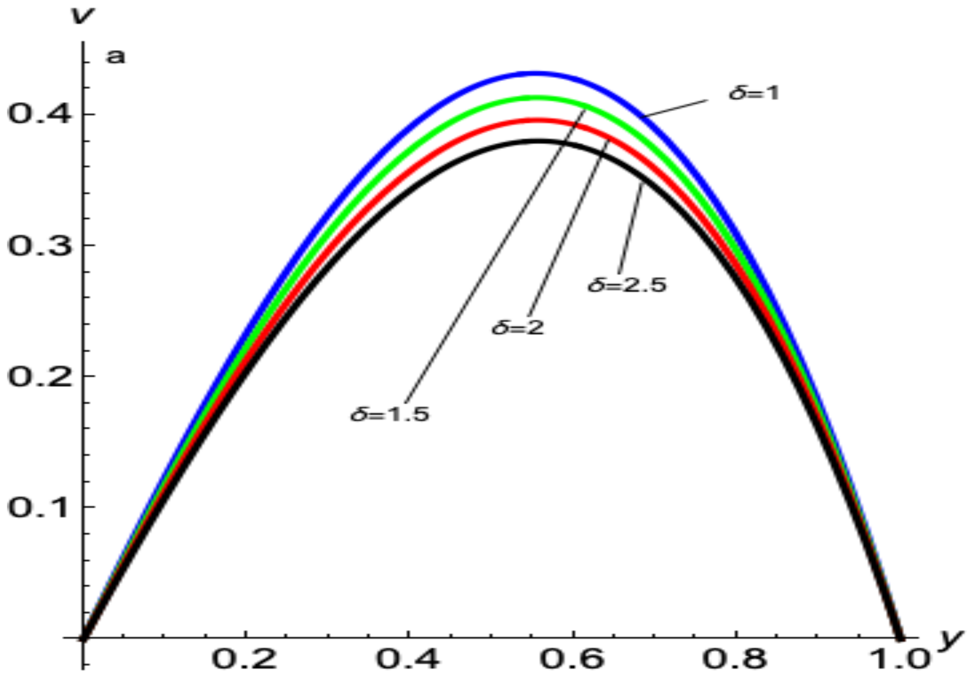


**Figure 4.11. A graph of velocity(v) against width(y) for varying values of viscous drag($\delta$).**

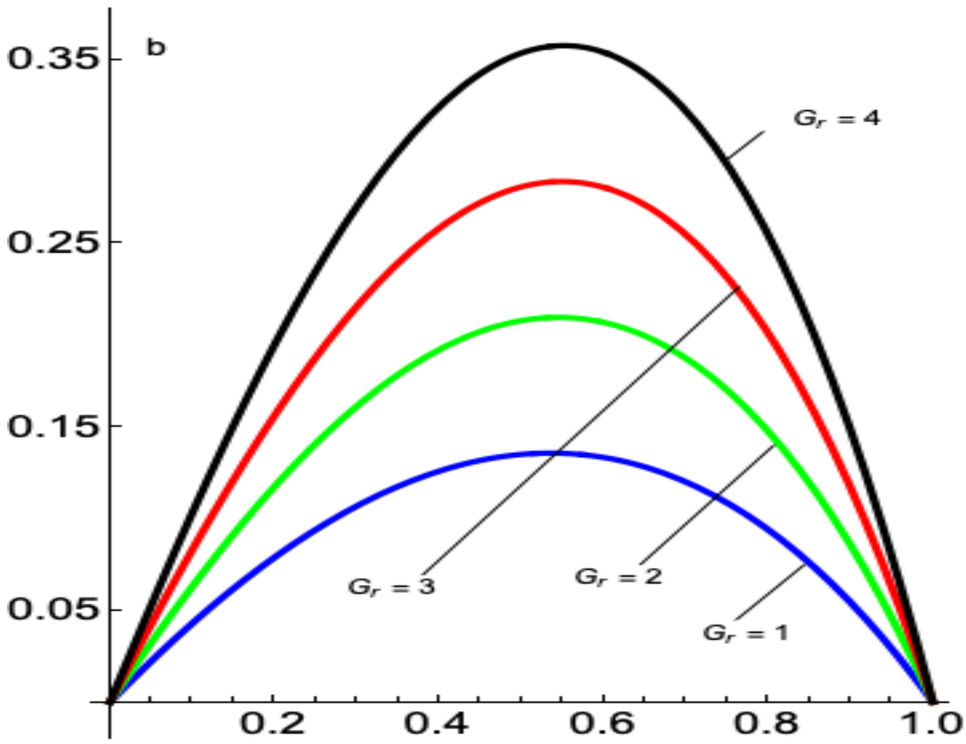


**Figure 4.12. A graph of velocity(v) against width(y) for varying values of Grashof number(Gr).**

Figures 4.7, 4.8, 4.9, 4.10, 4.11 and 4.12 are plots of the velocity of the fluid against the width of the material with varying values of ratio of the bulk fluid velocity on the solid displacement ($\eta$), heat source ($\alpha$), magnetic parameter (M), volume fraction of the fluid($\varphi$), viscous drag($\delta$), and Grashof number(Gr) of the fluid respectively. It was observed that every increase in the values of the ratio of the bulk fluid velocity on the solid displacement, magnetic parameter(M) and viscous drag($\delta$) resulted in the decrease of the velocity of the fluid. Also, for every increase in the values of the heat source ($\alpha$), volume fraction($\varphi$) and Grashof number(Gr), the velocity of the fluid increases.

### 4.1.3 Analysis of the Effects of the Control Parameters on the Temperature

The following graph represents the effect of the varying values of heat source ($\alpha$) on the temperature($\theta$) of the fluid.

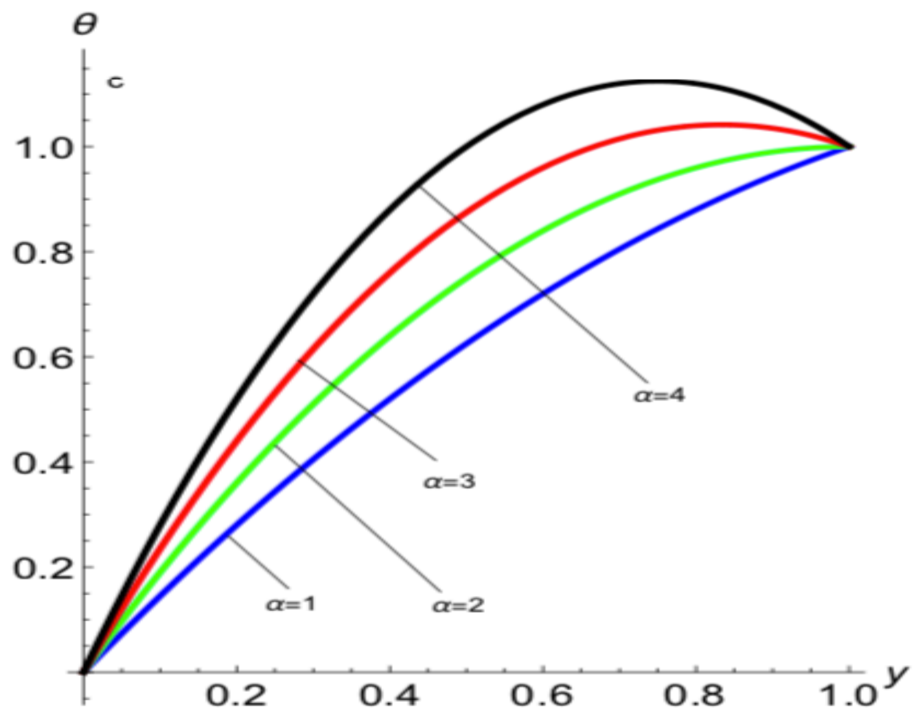


**Figure 4.13. A graph of temperature($\theta$) against width(y) for varying values of heat source ($\alpha$).**

Figure 4.13 is a plot of the temperature($\theta$) of the fluid against the width of the material with varying values of heat source ($\alpha$). It was observed that an increase in the heat source produces the heat, which enhances the temperature distribution across the MHD fluid.

### 4.1.4 Analysis of the Effects of the Control Parameters on the Entropy

The following graph represents the effect of varying values of the viscous dissipation parameter ($\frac{Br}{\Omega}$) on the entropy generation number($N_s$) of the fluid.

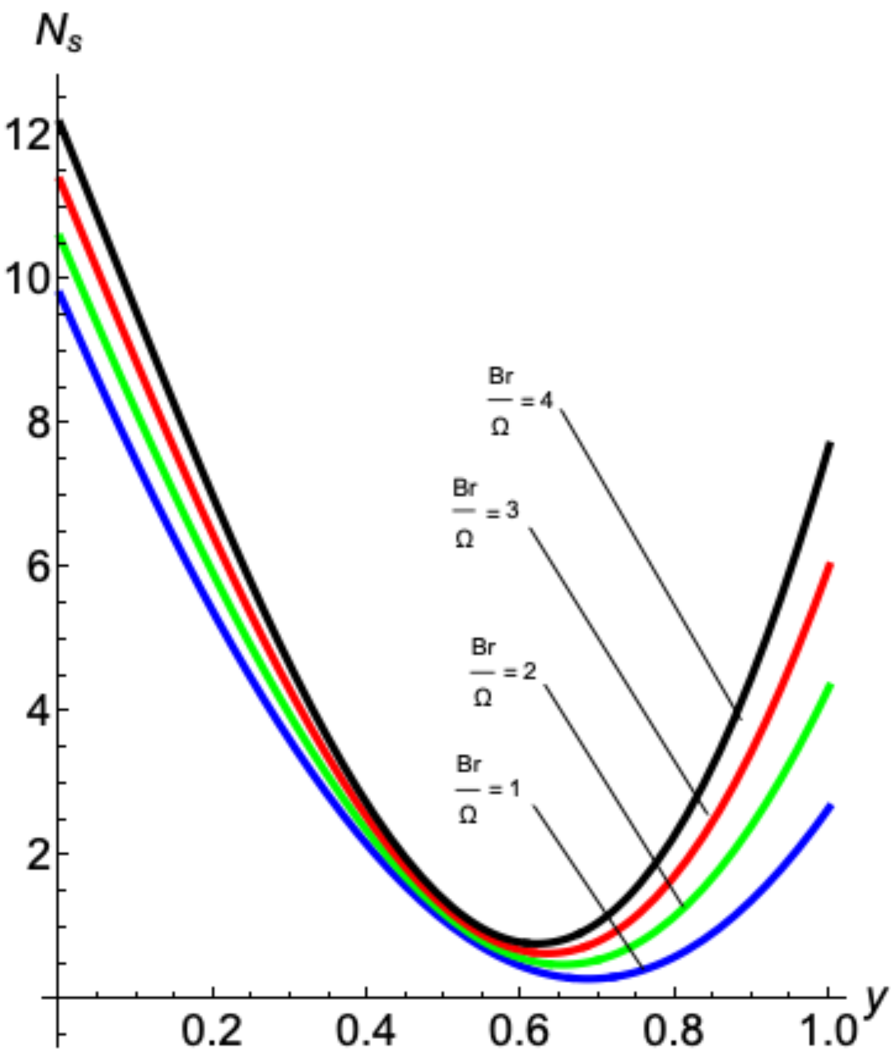


**Figure 4.14. A graph of entropy generation number($N_s$) against width(y) for varying values of viscous dissipation parameter ($\frac{Br}{\Omega}$).**

Figure 4.14 is a plot of the entropy generation number($N_s$) of the fluid against the width of the material with varying values of the viscous dissipation parameter ($\frac{Br}{\Omega}$). It was observed that an increase in the viscous dissipation parameter resulted in the increase of the entropy generation number of the MHD fluid.

# CHAPTER FIVE

# CONCLUSION AND RECOMMENDATION

## 5.0 Conclusion

This project analysed the effects of the changes of physical variables in the behaviour and the flow of the magnetohydrodynamic(MHD) fluid flowing in a vertical deformable porous media. The dimensional expression for the solid displacement, velocity, temperature and entropy generation number were obtained and then successfully converted to non-dimensional forms. Thereafter, the solutions of each non-dimensional form of the equations were solved using Adomian Decomposition Method(ADM) after applying the boundary conditions. The solutions to the equations obtained from the ADM were implemented into Mathematica which was then used to generate graphs for the solid displacement, fluid velocity, fluid temperature and the entropy to display the variations in the behaviour of the fluid flow under certain conditions.

A number of observations were also made during the course of the project, such as the velocity of the fluid flow increases with the viscosity ratio ($\eta$), and that the MHD fluid reduces in solid displacement as it becomes more magnetic and that an increase in the heat source produces the heat, which enhances the temperature distribution across the MHD fluid. It was also observed that the displacement of the fluid increases when there is an increase in the heat source ($\alpha$) of the fluid.

Based on the findings of the study, it can be concluded that the behaviour of an MHD fluid flowing in a deformable porous media at any time would be dependent on the various physical parameters in the fluid.

## 5.1 Recommendation

This project research would aid in further analysis of fluid flow in porous media, fluid flow of MHD and their applications in several industries and fields of study.

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