ANALYZING THE FLUTTER PHENOMENON IN AIRCRAFT: A REVIEW OF METHODS AND MODERN SOLUTIONS

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**A PROJECT SUBMITTED TO THE DEPARTMENT OF PHYSICS,
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**IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR THE
AWARD OF BACHELOR OF SCIENCE DEGREE IN PHYSICS.**

SEPTEMBER, 2022.

DECLARATION

I hereby declare that this project was written under the supervision of Dr. O.A. BABATUNDE, and is a product of my own research work. Information derived from various sources has been duly acknowledged in text and a list of references provided. This research project report has not been previously presented anywhere for the award of any degree or certificate.

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DATE

CERTIFICATION

This is to certify that the content of this project entitled “ANALYZING THE FLUTTER PHENOMENON IN AIRCRAFT: A REVIEW OF METHODS AND MODERN SOLUTIONS” was prepared and submitted by Donald Chidera Miracle with matriculation number 18010302002, in partial fulfillment of the requirements for the award of the degree of Bachelor of Science in Physics, Department of Physics of the Mountain Top University, Ogun state, Nigeria. The original research work was carried out by him under my supervision and is hereby accepted.

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Head of Department

DEDICATION

I dedicate this work to God Almighty for giving me the strength, and the grace to complete this project, and secondly my beloved parents, Mr. and Mrs. Donald for their guidance and support, the sacrifices they have made will forever be remembered in my heart.

ACKNOWLEDGEMENTS

I want to thank the Lord for His faithfulness upon my soul, I am forever grateful for His mercy and kindness over the time spent in the Mountain Top University. The years were gone in the blink of an eye as though I just started yesterday, only God could have made it go smoothly, Thank you Lord. Mr. and Mrs. Donald, I would love to show appreciation for all you have done for me over the years. The person I am today is all thanks to your grooming, and love for me all these years, God continue to bless you both, and increase his grace upon your lives. The project came out a success because I received guidance, the guidance came from my supervisor, Dr. O. A. Babatunde, he helped me put this work together, motivated me to complete the project, and better myself, thank you, Sir. The entire Physics Department also has my gratitude for giving me a place to belong in the school during my time here, it made adapting easier, as there was always a place to clear my head.

They ensured my academic journey reached this stage, provided me shelter, food, comfort and a home to return to, I want to appreciate my parents for their support throughout my journey as a student. The blessings of the Lord be upon everyone who has contributed to making my stay in this school a good memory, thank you all.

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ABSTRACT

In this project, the phenomenon of flutter wave occurrence in aircraft was carefully reviewed. The first sighting of flutter, how it was handled, and the evolution of aircraft design to accommodate flutter incidents were highlighted. The many methods of analyzing the flutter speed of an aircraft coupled with the software developed to help in simulating flight conditions which help determine when an aircraft will experience flutter were also presented. Lastly, the damping required to hamper the wild oscillation that occurs during flutter occurrences were also briefly enumerated.

The methods such as the k method, the p method, and the p-k method used in analyzing the flutter characteristics have been discussed. Also, a number of techniques used to damp the fierce oscillation during the flutter phenomenon in aircraft have been reviewed concisely. These include; the mass balance, aluminum lithium alloy, limiting the flight envelope, and the yaw damper.

The results showed that the shear stress of the aluminum lithium alloy increases with every rise in percentage of lithium. The rise in shear stress increases the stiffness of the material. The yaw damper greatly increases the response of a system to flutter. A system with a yaw damper after experiencing spontaneous flutter, has a setting time of 7.84 seconds. The system without a yaw damper has a setting time of 30.8 seconds, this is a large margin. The gap of 22.96 seconds is enough to damage a system unable to respond fast enough. The p-k method determines the degree of flutter at various speeds, calculating for any slight change during flight. The damping ratio of any system increases along with the mass ratio, large mass equals large damping in any oscillating system. The results reviewed show that flutter is significantly reduced using the mentioned solutions.

CHAPTER ONE

INTRODUCTION

1.0 Background to the study

The improvements experienced in the field of science have been occurring before most of the problems were given names. The foundation of these studies and solutions are rooted in the countless years our predecessors have used to discover new and interesting facts. This puts a relatable description to what is observed, and has been the driving force for all scientists and scholars over the years (Garrick and Reed III, 1981), this has led to the invention of equations that helped broaden our understanding and these equations have helped in the creation of many mechanical marvels and one of these marvels is the aircraft (Kundu, 2010).

The notion of the aircraft has existed for a while now in human history. Its origin can be traced back to the very early years of china, two thousand years ago when the kite was invented and used in their cultural festivals. They further improved their understanding and were able to develop lanterns that floated into the air. This would be the foundation for the hot-air balloon successfully built and tested in France during the eighteenth century. The exploits continued over the years, and discoveries were made about the forces that govern flight. Orville Wright and Wilbur Wright took innovation to a whole new level and made the first ever plane known to exist in the history of mankind in 1903. This invention became the basis for the popular design of the aircraft seen today. As our understanding of flight grew to a considerable amount, so did the variation of aircraft. Today more than 4 types of aircraft and other special modifications are existing (Watts *et al.*, 2012; McGovern, 2007). The main 4 are aerostats, aerodynes, fixed-wing, and rotorcraft.

The oldest and most popular of the aircraft is the hot-air balloon (Pfotzer, 1972), the envelope can be as tall as 20 feet tall. This is categorized as an aerostat, aircraft operating on the lighter-than-air principle. The method of flight is to heat the air in the envelope using a burner, this makes the air hot and less dense than the air in the atmosphere. The more you heat the air, the less dense it becomes the higher the altitude attained (Byrne, 2001). Hence the term aerostat, meaning it is an aircraft that attains flight by being lighter than the surrounding air. There are other examples of aerostats besides the hot air balloon, the blimps, and the biggest and most notorious, the Zeppelins. The Zeppelins and blimps can be termed dirigibles as they can be steered and have a power source.

The aerodynes need to generate lift using Newton's third law, by using downward thrust like the traditional rockets do. It generates thrust through the rocket engine, the jet engines can also be classified as aerodynes.

The fixed-wing is a more popular design in aircraft that imitates the physiology of birds. It also possesses a more efficient process of lift. The wings are constructed ensuring from the sideward perspective it takes the shape of an airfoil. The airfoil is a geometrical marvel that allows for the redirection of the air current to initiate lift in the aircraft. The predecessor that allowed for the growth of the design variation of the fixed-wing aircraft was the kite. The kite to date is still used to run certain tests. Exposing the conditions that determine a close hypothesis of how the fixed-wing aircraft would behave in real-life scenarios. (Kundu, 2010). They can be classified by their angle (either dihedral or anhedral) the wing support system, the number of wings, the wing platform and the position of the horizontal stabilizers if present at all.

In time a new breed would be born from all the experimentation and understanding, the rotorcraft. The lift is generated through the motion of the large aerofoil blades connected to the rotor. The rotorcraft has different forms; the helicopter, the autogyros, and the cyclogyros. There are other classifications of aircraft but these are the most practical examples in our world today.

There are other examples of aircraft such as the lift wing aircraft with the body designed to produce lift. The powered lift aircraft, a VTOL (a vertical take-off and landing) aircraft (Intwala, 2015). The Flettner plane generates lift using the Magnus effect, the ornithopter, and the wings flap as the wings of birds do generate lift. Aircraft are also categorized by their speed and size.

The Aircraft would not be possible if we did not possess knowledge of how airflow works when interacting with both static and dynamic structures. This study is called aerodynamics and our understanding of aerodynamics only got far because airflow was treated as fluid flow (Anderson Jr, 2007). This allowed for easy visualization of its interaction with the bodies it interacts with. Before aerodynamics there was the study of the movement of fluids. Fluid mechanics explores the world of dynamic behavior in fluids (fluid dynamics), and aerodynamics is a subject of study under fluid dynamics (Eckert and Michael, 2006). The link can be easily observed with a simple demonstration. The visibility of any observable phenomenon is key in understanding it better, as one of the main processes of observation is sight. Fluids transparency level is not a hundred percent but the same cannot be said for gas bodies. This demonstration begins with an erect object, an empty can on a medium-sized platform, a bottle of water, and a hand fan. When the bottle of water is splashed against the can, it pushes it backward or throws it off the platform completely. This supports the conclusion that the water has interacted with the body, this is not so different from when the hand fan is used. The point is to relate the splashing of water to the blowing of the fan. Both the fluid body(water) and the air molecules had the same effect on the can. Their density aside which would determine the force that needs to be applied for the desired result. The fluid bodies and gas bodies are not so different, hence the judge of air-flow as fluid-flow for better understanding. One could say that throwing a stone at the can also obtain the same result but the stone has a determined surface area of interaction as all solids do. The stone would not expand about the can during contact, which is exactly what both the fluid and gas bodies do.

The most significant fright of all existing life forms is the horror of extinction. As technology advanced, machines were built to make our lives easier but also kill faster, and one of those is the aircraft. Man has become able to fly but also exposed to death by crashing, considering the aggressiveness of the gravitational pull of our planet. Any slight accident could cause major damage.

There is a phenomenon called flutter, explained as one of the occurrences under dynamic aeroelasticity. Flutter occurs majorly in fixed-wing aircraft such as fighter jets, planes, and rockets. It is important to note that flutter also occurs in all structures that are exposed to airflow. It could be suspended structures such as bridges (Agar, 1989) or erected structures like very tall buildings in the form of skyscrapers and wild but beautiful architectural projects. Flutter is a widely experienced phenomenon in all structures interacting with air molecules, strong wind (gust) is an example of air molecules actively interacting with structures or vehicles. In 1916, the first ever recorded case of flutter was observed during the first world war. It was experienced by the tail wing which led to a deformation in the hull and the lopsided displacement of the tail wings asymmetrically. After this event, the pilot was consulted on his experience, and he suggested increasing the stiffness of the elevators using a strong shaft.

The event was further studied to reduce the probability of more unknown variables. This newly discovered variable capable of not just disrupting the flight stability (Freydin *et al.*, 2021) but also damaging the aircraft and putting the pilot in danger. The problems that may arise in future flight tests and actual flight sequences could be seen. This study continued and in 1926, Hans Reissner published a paper on the theory of wing divergence which shed light on the subject matter. By 1930, H. R Cox and A. Pugsley were in Farnborough at the RAE (Royal Aircraft

Establishment) when they termed the phenomenon ‘aeroelasticity’. The root of the official definition of aeroelasticity used today was a man named A. R Collar in the year 1947. A true understanding was formed after this definition was presented and the topic could be approached with better understanding from the view of anyone knowledgeable enough about forces and their interactions (Collar, 1978). The gap bridged by this newfound understanding was essential in laying a solid foundation for further extensive research.

It is observed in both dynamic and static structures. Aeroelasticity is evident everywhere if you are careful enough to pay attention (Manan *et al*., 2010). When the wind blows against the tree branches and causes them to move to and fro until eventually the branch is strained and breaks off. This is a good example of the process of aeroelasticity in structures. Flutter is researched and accepted as an occurrence under the presence of aeroelasticity in dynamic structures. Further categorized into hard and soft flutter (Ying *et al*., 2016), where net damping increases and decreases respectively.

1.1 Statement of the problem

The possibility of an aircraft crashing due to the flutter phenomenon is a serious issue plaguing the engineering world. This project is going over the various methods used to reduce flutter in aircraft, those already developed, and the methods currently in development. The aircraft experiences flutter especially on the wings, the activity of major forces like the aerodynamic forces, forces that determine flexibility, and the fictitious forces cause an unwanted oscillation at a particular frequency Looking at this interaction, a study on the old and recent contingencies created to reduce the effect of flutter in aircraft.

1.2 Aim and specific objective of the study

This project aims to give a level of understanding concerning the flutter phenomenon, explaining how it occurs, why it occurs, and how to stop it by going in-depth into the many solutions involved in the reduction of flutter during the structural design process of aircraft construction. This is to lay a foundation of our understanding so far, and also give a progress report on how far flutter has been dealt with. The specific objectives of this project are as follows:

i. define the phenomenon of flutter, its effect, and dangers to a knowledgeable extent.
ii. explain how flutter occurs in aircraft, what causes it, and when and where it occurs.
iii. list a good number of solutions already provided against the flutter phenomenon.

1.3 Purpose of the study

The purpose of this study is to create a sort of guideline that can be used in understanding the flutter phenomenon and its significance in the advancement of technology. The problem of flutter is dangerously underestimated. Believing that what has been seen so far is the extent to which flutter can affect structures is premature. History has proven that as the designs of our aircraft are modified, the flutter phenomenon affects it in a new way. There are always new parameters to consider for every modification made, leading to the necessary development of the knowledge gathered so far. This project is written to shed light on the existence of the flutter phenomenon, its simultaneous evolution with our technological advancement, mentioning the procedures already in place to reduce the effect on our aircraft.

CHAPTER TWO

LITERATURE REVIEW

2.0 A Review of the historical background of flutter

The heavens were considered the apex of conquest, chasing the heavens became a goal for humanity, allowing progress in the field of flight. There were stories in Greek mythology from the east, Icarus and Deadalus. The Wright brothers had a lot of inspiration from their predecessors, like Leonardo Da Vinci with his idea of the flying machine in the late 1400. The first time the word aeroplane was uttered, it was in a paper written by Joseph Pline suggesting a glider which is dirigible to be filled with gas and have a mounted propeller. In 1804 Sir George Cayley built a kite like glider with adjustable control surfaces, recorded as the first controllable aerodyne to remain airborne with ease. The aircraft kept developing over the years until the first major flutter scenario was recorded in 1916 (Kundu, 2010).

The flutter phenomenon has been in existence for years, mankind did not start to take notice until the early 90's around the time of the first world war. That does not mean flutter just started existing from then. The occurrence possessed realized importance, encouraging further study on it. Since its discovery, many experiments have been carried out to broaden the understanding on the topic. This is because of the danger flutter poses to any flying machine in existence. The natural forces of physics that govern our everyday lives have come together to manufacture a melody of disaster. A melody formed by the aerodynamic, fictitious, and elastic forces acting on the wings of any aircraft.

Firstly, the term flutter was not derived immediately nor identified as a solo entity. It was studied under a denomination it falls under, that denomination being aeroelasticity. This is ultimately defined as the interaction between the forces governing air-current, Inertia, and elasticity. Therefore, understanding flutter without understanding aeroelasticity is ill-advised.

Flutter being an occurrence under a type of aeroelasticity that is dynamic aeroelasticity (Blispinghoff *et al.*, 2013), deduced that aeroelasticity was studied first before flutter was studied in detail (Blispinghoff and Ashley, 2013). The types of aeroelasticity are dynamic and static aeroelasticity, each dealing with its specific study.

Static aeroelasticity deals with divergence and control reversal (Hodges and Pierce, 2011), the former being a situation where the twist of its elasticity becomes finite theoretically. This causes the wing to malfunction, and the latter being a phenomenon of reversed operation on wings with ailerons or any control surface. Dynamic aeroelasticity deals with studying the interactions between the three forces that have been mentioned frequently. This houses phenomena such as flutter, aeroservoelasticity, buffeting, propeller whirl flutter (Reed, 1967), and transonic aeroelasticity (Ashley and Holt, 1980). There have been many articles over the years on the study of aeroelasticity, different views, opinions, and discoveries over the course of its scientific history since discovery.

The aeroelasticity in a structure exists because of the interaction among the aerodynamic forces, elastic forces, and dynamic forces. The observation of aeroelasticity on the airplane is being viewed as a necessity; many incidents caused by flutter have led to serious structural damage and life-threatening events. The wings of the airplane interact the most with the aerodynamic forces; this is why the wings of an airplane or any aircraft experience flutter more than any other part of the aircraft (Blispinghoff *et al.*, 2013). The problem of flutter would not occur if the structures put in place when constructing the airplane were rigid. The phenomenon takes place when deformations of the structure add more aerodynamic forces. This addition produces extra structural deformations that impose even more aerodynamic forces, this exchange may become smaller or experience divergence and damage the structure.

The phenomenon of aeroelasticity, especially flutter and divergence could lead to failure in a structure. The design of aircraft needs to gain more weight to enable the integrity of the structure to remain, after being subjected to various adjustments in the stiffness and distribution of mass. The Handley-page 0/400 (bomber) in 1916 was the first recorded flutter event to be modeled and solved (Blispinghoff *et al.*, 2013). A well-detailed report of the developments in aeroelasticity and how it has influenced aircraft design can be seen in (Collar, 1978; Wright and Cooper, 2008).

When elastic structures are deformed during contact with an airstream, the resulting aerodynamic force is a by-product of the interaction. The structural dynamics of any body, deals with the forces that act on the body when it is in a resting state (Hodges and Pierce, 2011). It is important to note that the study of the dynamics in a structure has helped in more ways than one when it comes to understanding how it is flutter causes so much damage, the study of flutter as a dynamic aeroelastic phenomenon goes hand-in-hand with the study of the structure dynamics. The dynamics of a structure discusses the forces of inertia in that body (Hodges and Pierce, 2011).

Flutter has caused a rapid evolution in aircraft design. This is because failing to improve on the existing designs will cause a catastrophic rise in flutter incidents. This would lead to major damage in air transport system and cause the development of the skies as another unexplored frontier. The times have come and gone but the tenacity of man towards eradicating this problem is evident (Garrick and Reed III, 1981). Our predecessors have laid the groundwork for further research and in this modern age, we continue to find innovative ways to eliminate the terror that is the flutter phenomenon.

Many principles that conduct the activities of aeroelastic occurrences are diligently studied to the extent we can. Having noticed where it is most prominent in flying vehicles, that is the control surfaces, the wings, and its propulsive tendencies. Because of the consequences on a large scale, flutter has gained a notoriety in the flight industry. A study finalized in the year 1956 shows that during a five-year period preceding the finalization of the study carried out, the military aircraft were subject to incidents of flutter (Blispinghoff and Ashley, 2013).

Furthermore, various testing methods to determine the conditions in which flutter occurs in an aircraft. The algorithms for identifying parameters used to locate frequency and estimate damping with the response data, processing of digital data, excitation in structures, and systems for instrumentation are all explained. Experiences from real-life events, and test program show the general efficiency of the many methods used, these results encourage a brighter prospect for developments in the future (Kehoe, 1995).

The diagram in Fig 2.0 shows the interaction between forces in aeroelasticity.

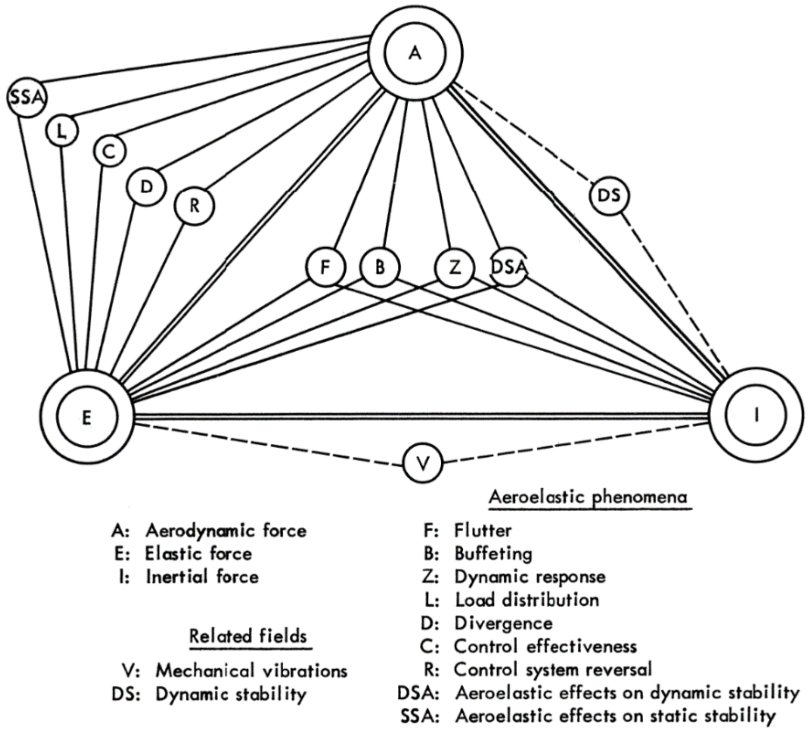

Fig 2.0. Interaction of forces during aeroelasticity (courtesy of Blispinghoff *et al.*, 2013).

2.1 A Review of the theoretical background of Flutter

The laws and forces involved in aeroelasticity are grounded in physics, possessing theories and equations that explain how they operate, and how they are calculated. The Collar diagram in Fig 2.1 shows the interaction between the three major forces involved in aeroelasticity. We will take the opportunity to understand the theory behind these forces and how they are related to the flutter phenomenon in aeroelasticity.

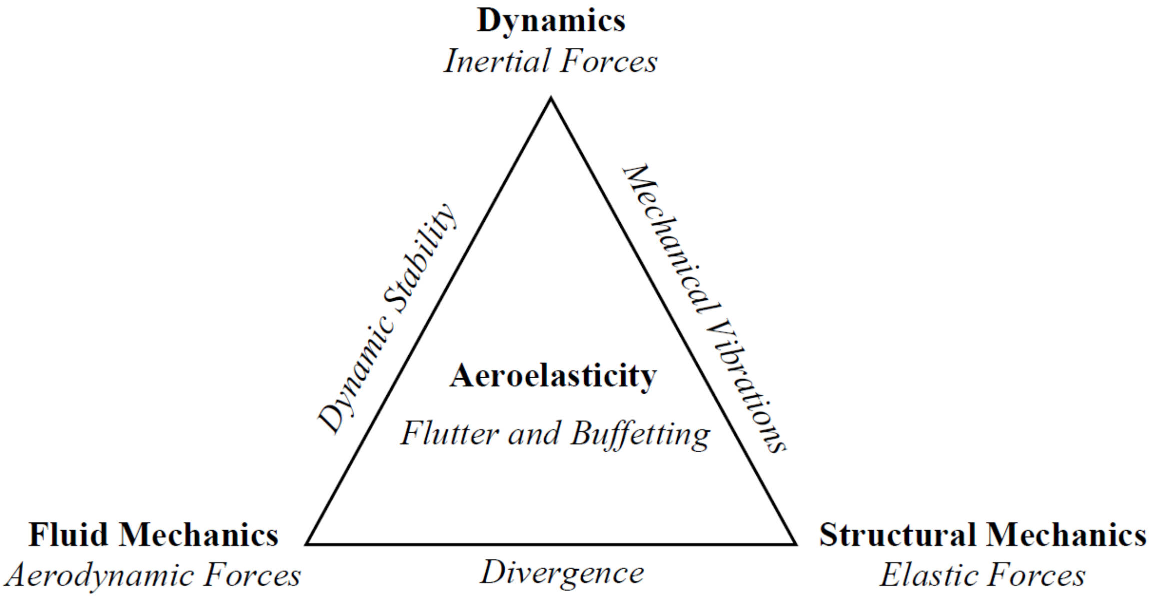

Fig 2.1 Collar’s triangle of aeroelasticity.

Aeroelasticity as a denomination is built on many laws and theories both official and unofficial. The focus will be on the official theories explaining the three primary forces involved, and showing in both words and equations the parameters involved in the activity of the forces. The elastic forces, the aerodynamic forces, and the inertial forces, their mathematical representation are as displayed below:

2.2.0 Aerodynamic Forces

There are four principles by which aerodynamics act upon a structure, Lift, Weight, Thrust, and Drag. These forces are responsible for the commencing of flight, the sustaining of flight, and the landing after a flight. The mathematical formulas are as follows:

$$\textbf{\textit{Lift}} = C_l \times \frac{e \times v^2}{2} \times A \qquad (2.1.1)$$

Where; C_l is the lift coefficient, e is the density, v^2 *i*s the squared velocity*,* and A is the reference area.

$$\textbf{\textit{Thrust}} = V \frac{dm}{dt} \qquad (2.1.2)$$

Where; V is the velocity, *dm* is the change in mass*,* and *dt* is the change in time.

$$\textbf{\textit{Weight}} = m \times g \qquad (2.1.3)$$

Where; W is the weight, m is the mass, and g is the acceleration due to gravity.

$$\textbf{\textit{Drag}} = C_d \times \frac{\rho \times v^2}{2} \times A \qquad (2.1.4)$$

Where; C_d is the drag coefficient, ρ is the density of air, v^2 is the squared velocity, and *A* is the reference area.

2.3.0 Inertial Force

The original law of inertia was first mentioned in the context of horizontal displacement on earth by Galileo Galilei but was later broadened to all planes by Rene Descartes. This made the possibility of calculating inertia for all bodies a brighter prospect. It describes the forces acting on a body when it is in a static state, dynamic state, and when it is transitioning from static to dynamic disposition.

The inertial forces operate on Sir Isaac Newton's three laws of motion. These laws have become a part of the foundation of classical physics, allowing for an understanding of how bodies behave in a state of rest and equilibrium. Further expanding on his understanding, the laws were adapted into equations which became a normal practice in physics.

2.3.1 Newton's First Law

It states that a body retains a static disposition unless externally influenced. In motion, a body continues on its path at a stable level of acceleration unless interacted with by an external body.

2.3.2 Newton's Second Law

The change in momentum is equal to the magnitude and direction of the force exerted on the body. Mathematically depicted as:

$$F = ma \qquad (2.1.5)$$

Where; F is the force, m is the mass, and a is the acceleration.

2.3.3 Newton's Third Law

This law states simply that for every or any exertion of force upon any surface, there is a reciprocal reaction of similar force. In other words, for all actions, there is an equal and resistive reaction.

2.4.0 Elastic Force

The elastic force present in a body represents the reversing of the deformation that has occurred in a system. It is responsible for returning a body to its original shape after force has been applied to deform the geometry. The young modulus, the modulus of elasticity, Tensile stress, and tensile strain are all examples of parameters considered when determining elasticity in a body or system. The following formulas are a basis used to calculate the necessary information needed about a system's elasticity. The general formula for elastic force in a body is:

$$\vec{F} = -k\Delta\vec{x} \qquad (2.1.6)$$

Where; $\vec{F}$ is the force, $-k$ is the Hooke's constant, and $\vec{x}$ is the stretch or contraction.

Furthermore**,** under elasticity, the parameters observed in a system that allows for experimentation and helps calculate the elastic modulus of the system are, stress and strain.

2.4.1 Stress – The force of each unit area in a material exerted by external forces, asymmetrical heating, or perpetual deformation which enables accurate understanding, and predicting of elasticity in all bodies (solid, liquid, and gas).

2.4.2 Strain – The level of deformation in a body parallel to the applied force concerning the original measurements of the body.

2.4.3 Young modulus – The ability of a material to withstand the changes in length when placed under tension and curtailment.

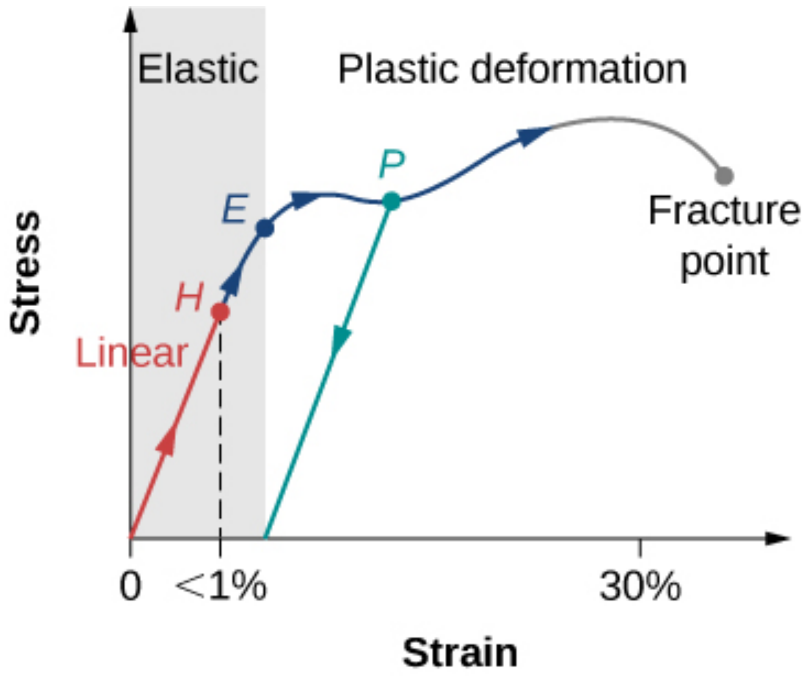

Fig 2.1.1. Graph of elasticity in physics.

Young Modulus

$$\boldsymbol{E} = \frac{tensile\ stress}{tensile\ strain} \qquad (2.1.7)$$

Modulus of elasticity

$$\lambda = \frac{stress}{strain} \qquad (2.1.8)$$

Tensile strain

$$\varepsilon = \frac{e}{L} \qquad (2.1.9)$$

Where; e is the extension in the system, and L is the original length of the system.

Tensile stress

$$\sigma = \frac{F}{A} \tag{2.2.0}$$

Where; F is the force, and A is the area.

The law that is generally accepted as an accurate representation of how elasticity function's Hooke's law. Hooke's law states that the force required to lengthen or constrict an object is scaled linearly to that distance, meaning that for any relatively small deformation on an elastic structure, the strain is proportional to the stress. The formula that depicts Hooke's law is seen in 2.1.6.

2.5.0 Theoretical formulation of methods used in flutter analysis

A formula exists to predict flutter in aircraft, it is not so much as predicting flutter as it is more like determining the exact conditions flutter would occur in an aircraft. The aircraft is represented by a mathematical equation known as the mathematical model of the aircraft, then it is put into a formula to calculate flutter speed. There are also methods of analyzing flutter, the p-method, the k-method, and the p-k method. The k-method is an unreliable method of analyzing flutter due to how realistic it can be. It predicts flutter considering the presence of a bit of structural damping, as opposed to the p-method used to predict flutter in a structure with zero damping.

2.5.1 The problem

$$\begin{bmatrix} 1 & x_\theta \\ x_\theta & r^2 \end{bmatrix} \begin{Bmatrix} \ddot{\bar{h}} \\ \ddot{\theta} \end{Bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & \sigma^2 r^2 \end{bmatrix} \begin{Bmatrix} \bar{h} \\ \theta \end{Bmatrix} = \begin{bmatrix} 0 & -\frac{2V^2}{\mu} \\ 0 & \frac{2V^2}{\mu}\left(\frac{1}{2} + a\right) \end{bmatrix} \begin{Bmatrix} \bar{h} \\ \theta \end{Bmatrix} \tag{2.2.1}$$

We will solve these problems using the p and k method of flutter analysis

2.5.2 P – method

$$\lambda^2 \begin{Bmatrix} \hat{h} \\ \hat{\theta} \end{Bmatrix} = \begin{bmatrix} 1 & x_\theta \\ x_\theta & r^2 \end{bmatrix}^{-1} \begin{bmatrix} -1 & -\frac{2V^2}{\mu} \\ 0 & -\sigma^2 r^2 + \frac{2V^2}{\mu}\left(\frac{1}{2}+a\right) \end{bmatrix} \begin{Bmatrix} \hat{h} \\ \hat{\theta} \end{Bmatrix} \qquad (2.2.2)$$

2.5.3 K – Method

$$\frac{1+ig}{\omega^2} \begin{Bmatrix} \hat{h} \\ \hat{\theta} \end{Bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & \sigma^2 r^2 \end{bmatrix}^{-1} \begin{bmatrix} 1 & x_\theta - \frac{2}{\mu k^2} \\ x_\theta & r^2 + \frac{2}{\mu k^2}\left(\frac{1}{2}+a\right) \end{bmatrix} \begin{Bmatrix} \hat{h} \\ \hat{\theta} \end{Bmatrix} \qquad (2.2.3)$$

2.5.4 P-K method

The p-k solution for flutter analysis is a measure against flutter. A method used to derive a true damping solution, by giving approximate true damping results used routinely for flutter analysis, and flutter tests during flight.

The equation for the p-k method for modal flutter analysis

$$\left[M_{hh}p^2 + \left(B_{hh} - \tfrac{1}{4}\rho \dot{c} V Q^I_{hh}/k\right)p + \left(k_{hh} - \tfrac{1}{2}\rho V^2 Q^R_{hh}\right)\right]\{u_h\} = 0 \qquad (2.2.4)$$

The new terms are as follows:

Q^R_{hh} = aerodynamic damping modal matrix, a function of Mach number, m, and reduced frequency, k

Q^I_{hh} = aerodynamic stiffness modal matrix, a function of Mach number, m, and reduced frequency, k

ρ = The eigenvalue = $\omega\ (\gamma \pm i)$

γ = transient decay rate coefficient (Note that the structural damping coefficient $g = 2\gamma$)

2.6.0 Wave equations

The wave equations for the various levels of damping in a system or structure are shown below:

Zero damped

$$x(t) = Ae^{-ct/(2m)} \sin(\omega_d t + \emptyset) \qquad (2.2.5)$$

Critically damped

$$x(t) = (A + Bt)e^{-wt} \qquad (2.2.6)$$

Overdamped

$$x(t) = Ae^{\lambda_1 t} + Be^{\lambda_2 t} \qquad (2.2.7)$$

The damping ratio of a system represents the decay of the oscillation after a disturbance.

Damping ratio

$$\zeta = \frac{C}{C_r} \qquad (2.2.8)$$

Where; ζ is the damping ratio, C is the damping coefficient of the system, and Cr is the critical damping coefficient.

Newton's second law for forced, damped oscillation

$$= -kx - b\frac{dx}{dt} + F_0 \sin(\omega t) = m\frac{d^2x}{dt^2} \qquad (2.2.9)$$

The solution to **2.2.9** is

$$x(t) = A\cos(\omega t + \phi) \qquad (2.3.0)$$

The amplitude of a system undergoing forced, damped oscillation

$$= \frac{F_0}{\sqrt{m\left(\omega^2 + \omega_0^2\right)^2 + b^2\omega^2}} \qquad (2.3.1)$$

These are the wave equations for the different levels of damping in a system, commonly used when calculating the frequency of that system during tests and observations.

The mode by which the wings oscillate during flutter can be represented by a wave equation. The wave equations differ based on the degree of damping in the equation. The possible wave equations are as follows:

2.6.1 Elastic waves

They travel generally through fluids, and their surface, leaving behind no noticeable changes to the structure. They move through water and air, and energy travels through solid materials. For example, how heat moves through the earth.

$$\rho \ddot{u} = f + (\lambda + 2\mu)\nabla(\nabla . u) - \mu\nabla\times(\nabla\times u) \qquad (2.3.2)$$

Where:

$\lambda \; and \; \mu$ are the Lamé parameter defining the elastic properties of the structure.

ρ is the density**.**

f is the function of the source and u is the vector of displacement.

2.6.2 Mechanical waves

Mechanical waves exist only in materials with elasticity and inertia. When matter oscillates and ends up producing a transfer of energy through mediums, the medium’s range is restricted, hindering it from straying far from its equilibrium position of origin. Mechanical waves are classified into transverse waves, longitudinal waves, and surface waves.

2.6.3 Flutter speed

The base formula representing speed is the change in distance divided by the change in time, mathematically represented as;

$$\textbf{Speed} = \frac{change\ in\ distance}{change\ in\ time} = \frac{\partial D}{\partial t} \qquad (2.3.3)$$

The aircraft design and flight sequences are taken into account and converted into a mathematical model to fit into the equation used to calculate flutter speed. We will be looking at the mathematical model for aircraft in its basic form. The full description can be looked up in the article "***mathematical model for control of aircraft and satellites***" by Thor I. Fossen in 2011. Starting with the aircraft state-space vectors such as the velocity vectors.

2.6.4 Aircraft State-Space Vectors

$$v := \begin{bmatrix} U \\ V \\ W \\ P \\ Q \\ R \end{bmatrix} = \begin{bmatrix} longitudinal(forward)velocity \\ lateral(tranverse)velocity \\ vertical\ velocity \\ roll\ rate \\ pitch\ rate \\ yaw\ rate \end{bmatrix} \qquad (2.3.4)$$

$$\eta := \begin{bmatrix} X_E \\ Y_E \\ Z_E, h \\ \Phi \\ \Theta \\ \Psi \end{bmatrix} = \begin{bmatrix} Earth - fixed\ x - position \\ Earth - fixed\ y - position \\ Earth - fixed\ z - position(axis\ downwards), altitude \\ roll\ angle \\ pitch\ angle \\ yaw\ angle \end{bmatrix} \qquad (2.3.5)$$

There are also the aerodynamic forces and Moments represented by the aerodynamic coefficients shown below

$$X_{index} = \frac{\partial X}{\partial index} \qquad (2.3.6)$$

$$Y_{index} = \frac{\partial Y}{\partial index} \qquad (2.3.7)$$

$$Z_{index} = \frac{\partial Z}{\partial index} \qquad (2.3.8)$$

$$L_{index} = \frac{\partial L}{\partial index} \qquad (2.3.9)$$

$$M_{index} = \frac{\partial M}{\partial index} \qquad (2.4.0)$$

$$N_{index} = \frac{\partial N}{\partial index} \qquad (2.4.1)$$

The standard aircraft maneuvers are represented mathematically as:

Straight flight: $\Theta_0 = Q_0 = R_0 = 0$ (2.4.2)

Symmetric flight: $\Psi_0 = V_0 = 0$ (2.4.3)

Flying with wings level: $\Phi_0 = P_0 = 0$ (2.4.4)

Steady turn: $\mathrm{R}_0 = constant$ (2.4.5)

Steady pitching flight: $Q_0 = constant$ (2.4.6)

Steady rolling/spinning flight: $\mathrm{P}_0 = constant$ (2.4.7)

Mathematical model of the aircraft “F3B Bristol Fighter”

The aircraft identified with the British in the first world war. The model is for a calculated turn where **β = 0**. The vector of state-space is depicted as:

The arrangement of the values of the state space vectors is exactly as depicted in 2.1.9**.**

$$X_{lat}\begin{bmatrix} p(deg/s) \\ r(deg/s) \\ \phi(deg) \\ \psi(deg) \end{bmatrix}, \boldsymbol{u}_{lat} = [\delta_A(deg)] \qquad (2.4.8)$$

The equation that calculates the flutter velocity is shown below:

Practically speaking, it is necessary to calculate for U with M fixed to determine the stability. We show that the modes are zeros of the function

$$d = (M, \lambda, U) \qquad (2.4.9)$$

For U = 0 we derive the modes of the structure, bending, and pitching, we start with

$$\lambda_k = (\lambda, M, 0) = i\omega_\kappa \qquad (2.5.0)$$

These are the modes of the structure, and we depict that:

$$\frac{\partial}{\partial\lambda}\, d(M, \lambda, U)\Big|_{\lambda = i\omega_\kappa} \neq 0 \qquad (2.5.1)$$

We do this to define the roots $\lambda_k(M, U)$ as a function of U using

$$-\frac{\frac{\partial d}{\partial U}}{\frac{\partial d}{\partial \lambda}} = \frac{\partial\lambda(M,U)}{\partial U} \qquad (2.5.2)$$

For example,

$$\lambda_k(M, 0) = i\omega_k \qquad (2.5.3)$$

This is the kth mode of bending we keep identifying $\boldsymbol{\lambda_k(M, U)}$as the root locus. Let

$$\sigma_K(M, U) = Re\lambda_k(M, U) \qquad (2.5.4)$$

Then,

$$\sigma_k(M, 0) = 0 \qquad (2.5.5)$$

The curve of $\sigma_K(M, U)$ is called the "***stability curve***". We show that:

$$\frac{\partial\sigma_k(M,U)}{\partial U}\Big|_{U=0} = constant\ \left(\frac{-1}{M}\right) \qquad (2.5.6)$$

For all k with constant whether the mode is pitching or bending, enabling us to identify the first-time flutter speed.

The Flutter speed equation is given as:

$$U_F(M) = \sigma_K(M, U) = 0, \frac{\partial\sigma_K(M,V)}{\partial U} > 0 \qquad (2.5.7)$$

CHAPTER THREE

METHODOLOGY

3.0 Introduction

The problem of flutter warranted a solution, and over the years many have been discovered, tested, and applied to limit the effect of the flutter phenomenon on aircraft. Some of these solutions are mass balances, dampers, the creation of alloys with higher material stiffness, limiting the flight envelope, and increasing eigenfrequencies by reducing the mass or increasing the stiffness. Methods of flutter analysis also exist to solve the problem of flutter, we have 3 major ones, the k-method, the p-method, and the p-k method.

3.1. Mass Balance

The mass balance is applied to the control surfaces of aircraft mass balances work by balancing an unknown mass with a known mass. It uses the law of conservation during the analysis of the system depending on the problem, but everything is based on the conservation of mass (Himmelblau and David. 1967). When the wing is experiencing flutter at the control surfaces, the control surfaces of an aircraft can be deformed so they possess high elasticity and low stiffness to allow for easy manipulation but this also makes it an easy target for the flutter phenomenon to act upon. The mass balance will be attached to the control surface to cancel the unknown mass generated by the force working against the wing by opposing it with a presently known mass, the mass balance would have been a permanent solution to flutter if not for its weak strength. Fig 3.1.0 is an example of how a mass balance is installed on an aircraft. The balance condition is displayed in Fig 3.1.1**.** Further illustrated in Fig 3.1.2**,** the mass balance adjusts the center of gravity on the wing.

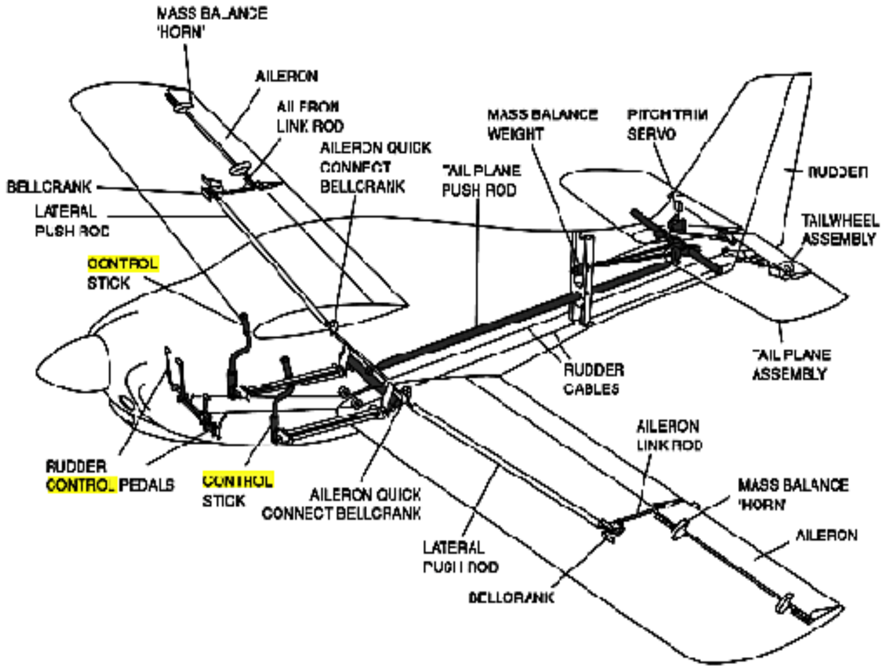

Fig 3.1.0. Mass balance position on a plane.

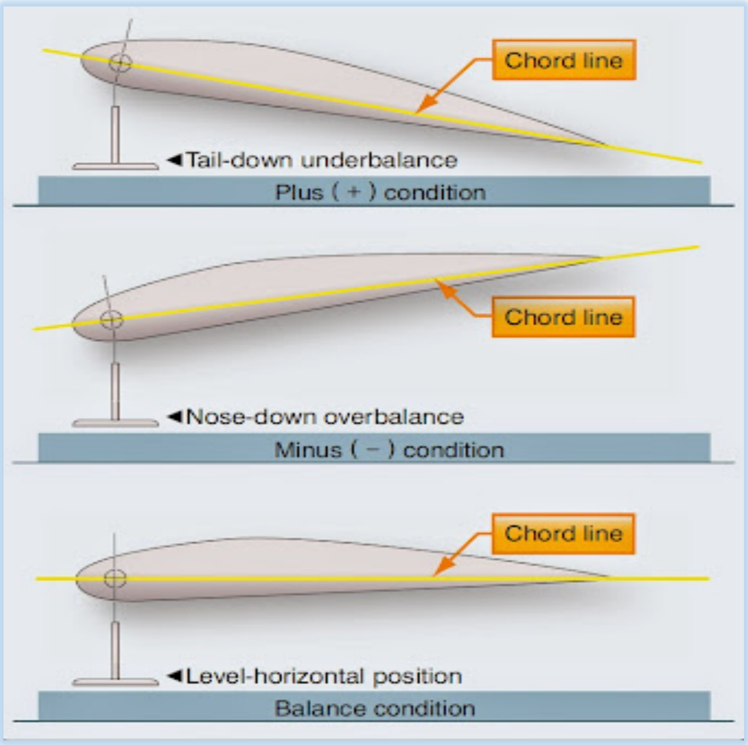

Fig 3.1.1. Balance condition of a wing during flight.

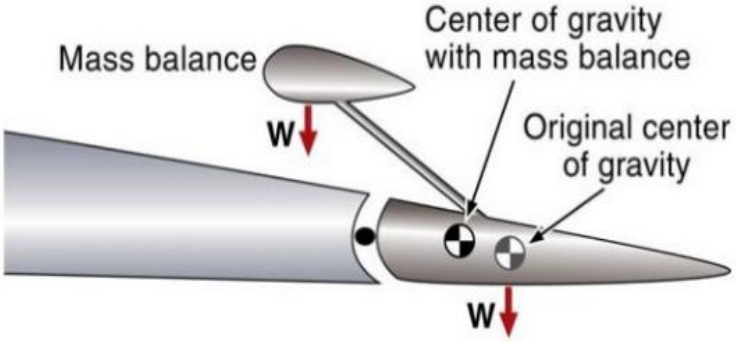

Fig 3.1.2. Mass Balance.

3.2. Aluminum-Lithium alloy

The strongest material with minimal weight and high strength in existence currently is carbon fiber, it has high strength and flexibility ratio, enabling deformation without the fear of breakage. The issue with using carbon fiber to create a plane is the high cost, to find a way around this effective but expensive material, scientists moved to the creation of an alloy that would perform just as well, and finally, they created the aluminum alloy called aluminum-lithium. This alloy is made up of 80% aluminum and 20% lithium. It has high strength, meaning the stiffness is high as well, it has the benefits of aluminum retained as well since aluminum is the best metal for making aircraft. It is still in its growing phase and not yet as commercially accepted as carbon fiber but it has the potential to be as efficient as carbon fiber in combating flutter.

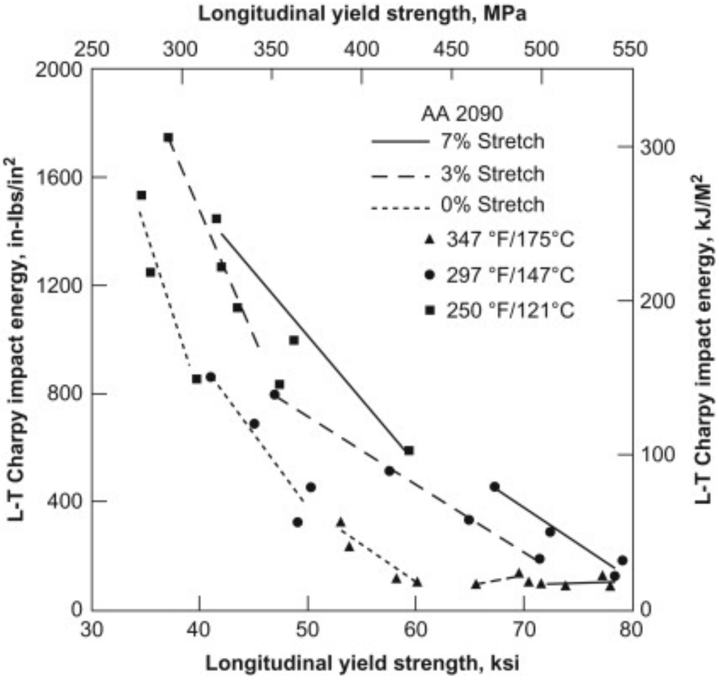

Fig 3.2.0. Effect of stretch and temperature on the strength-stiffness.

The yield strength can be viewed as the visible observation of the elastic limit of a material, deformation occurs once the yield strength has been surpassed. The figure above shows the effect of temperature and stretching on the alloy.

3.3. Flight envelope limit

The flight envelope is the altitude at which the aircraft can comfortably operate in, any unforeseen event can easily react efficiently since the aircraft is still within the region its body can handle (Poulos and Lindley, 2014). Limiting the envelope is deciding to set a restricted area for the aircraft within its region of operation. In every aircraft's flight envelope there is an altitude where the air currents get wilder and stronger, increasing the probability of countless interruptions especially flutter which occurs when the fictitious, elastic and aerodynamic forces are at their base limit, and reaching their breaking point. The safest zone within the flight envelope should be the limit to completely reduce the probability of flutter occurring unexpectedly. The zone with the highest level of atmospheric probability in which the parameters involved in flight are within expectations and even if they do exceed expectation, it is within an adjustable margin.

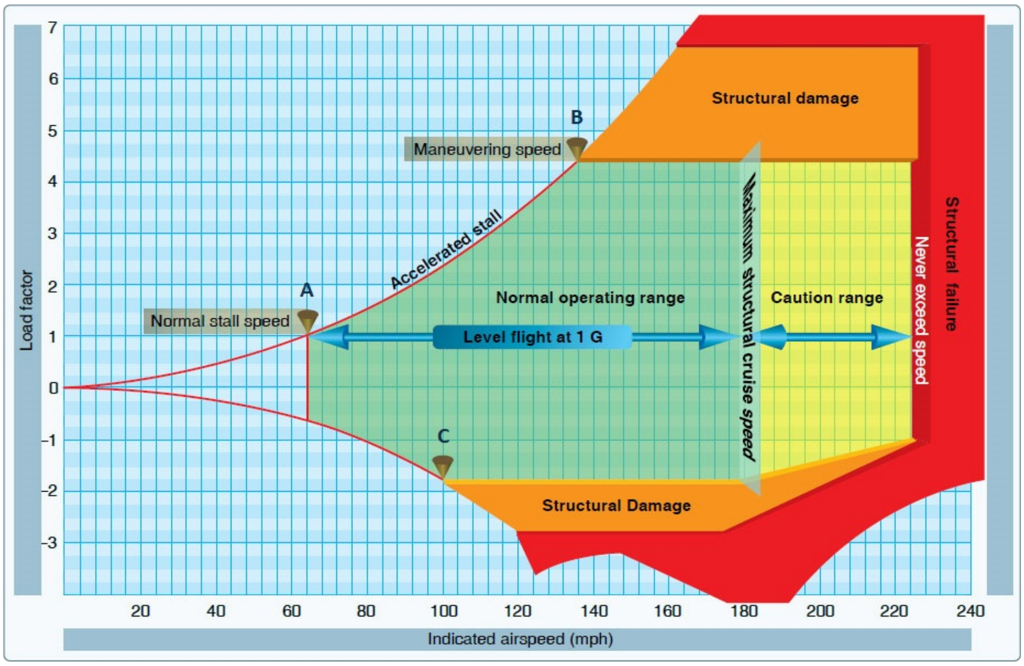

Fig.3.3.0. Flight envelope range classification of aircraft (courtesy of www.uavnavigation.com)

3.4. Yaw Damper

Dampers are mechanisms used to maintain a level of equilibrium in any system they are installed in (Zhao *et al*., 2021), it comprises actuators and dashpots arranged to reduce mass displacement in a system. The dashpot works using the physics of hydraulics. The Yaw damper is the common damper used on the control surfaces of airplanes, though it is mainly used to stop the plane from yawing unnecessarily, it is also a good measure against flutter on the wings of the aircraft.

Fig 3.4.0 Actuators used in Yaw Damper system set-up.

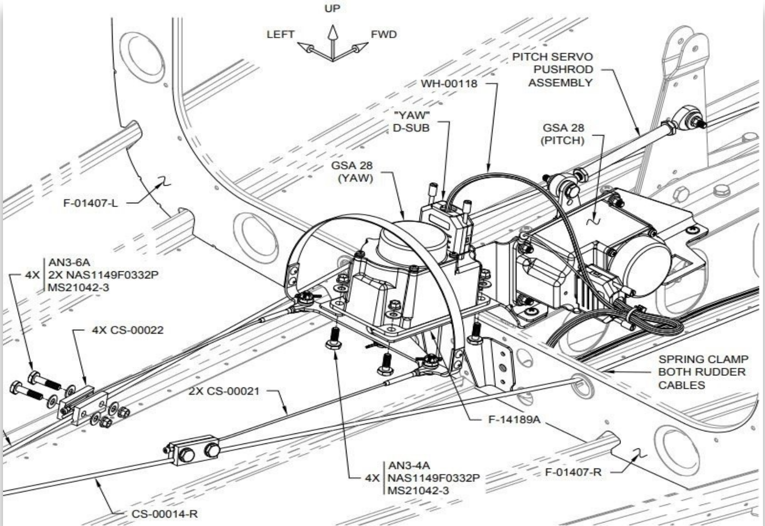

Fig 3.4.1. Yaw Damper Servos and Cables.

The term yaw damper is regarded as a misnomer (Schiff and Barry, 2010), meaning the name does not properly describe the object it is attached to. A yaw damper does not necessarily stop yawing of an aircraft, as yawing is necessary for the maneuverability of the aircraft. The yaw movement on an aircraft is the turning of the aircraft to the left or the right using the rudder. The yaw damper possesses various accelerometers and high-end sensors which observe the frequency of yaw in the aircraft, connected to the system that controls flight, it sends signals to the rudder control surface when incidental yaw is observed, adjusting the rudder to maintain a stable motion in the aircraft (Mark and Rob, 2017). The unexpected yawing is called slipping or skidding. The yaw damper can be connected in series or parallel. The installation of the dampers in series is illustrated in Fig 3.4.3, and that of the parallel set-up is shown in Fig 3.4.2.

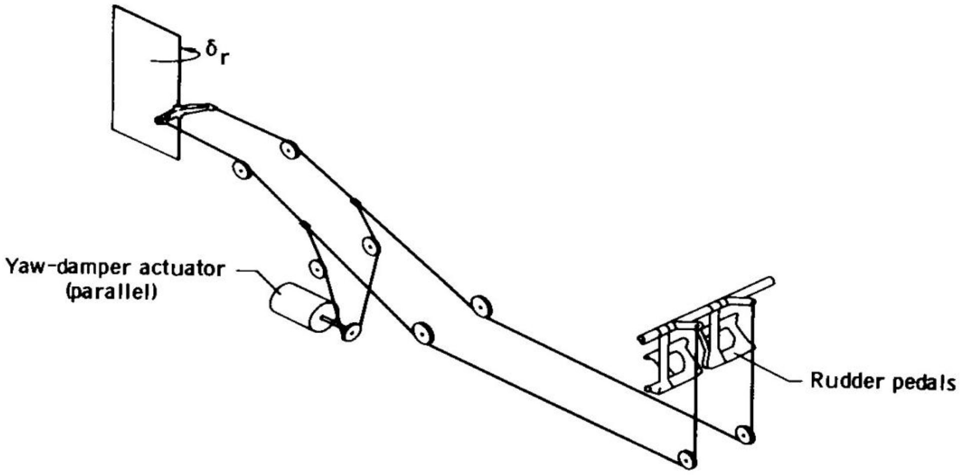

Fig 3.4.2. A parallel system of Yaw Damper.

The parallel system yaw damper is connected to the control cords which are also connected to the pedals. The control cords are connected to the rudder, enabling the influencing of the rudder of the aircraft. The yaw damper acts as a scale to balance the aircraft when incidental yaw occurs. When the pilot is initiating the yaw, the damper does not influence the rudder as the yaw is not incidental. The yaw damper can differentiate between initiated yaw and incidental yaw through the sensors and accelerometers connected to the control system of the aircraft.

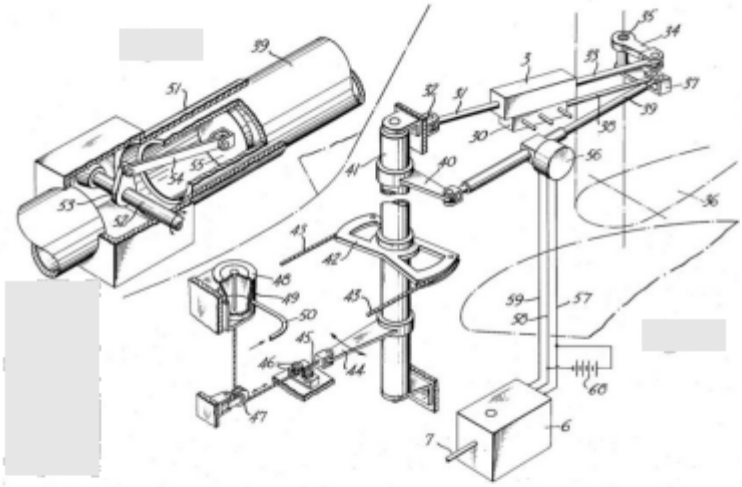

Fig 3.4.3. Series Yaw Damper for the Boeing 707.

The series system operates sequentially, starting by sending the yaw rate data (from the rate gyro or the bob weight) to a mechanical motor that changes the length of the control pole that commands the hydraulic actuator in turn moving the rudder. The extending and retracting process of the pole is how the activities of the rudder are not relayed to the cables.

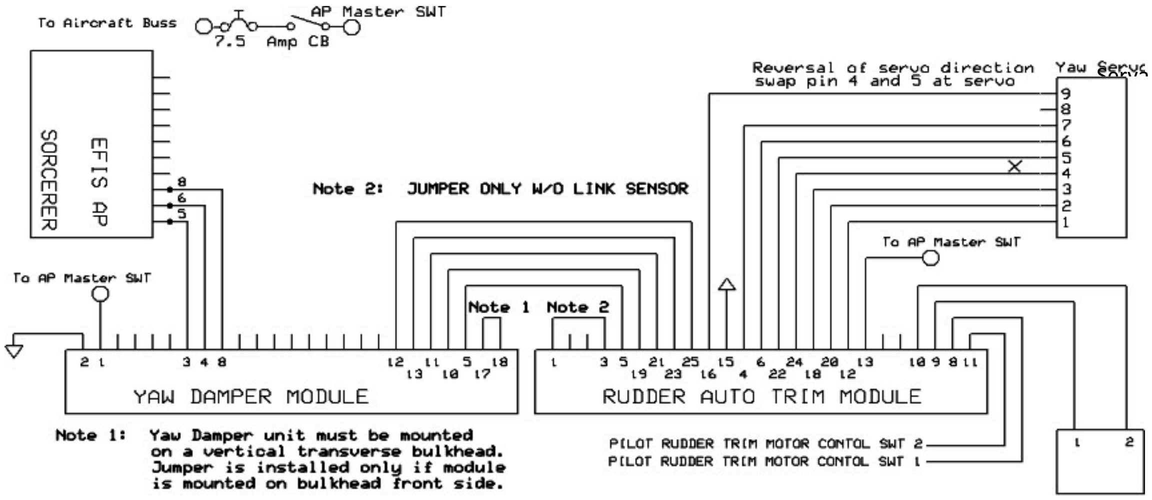

Fig 3.4.4 Circuit installation of Yaw Damper unit mounted on a vertical transverse bulkhead with the jumper only installed with the module mounted on the front of the bulkhead.

Table 3.4.5. The troubleshooting process in Yaw Dampers.

Problem	Cause	Corrective Action
Yaw Servo will not move and is quiet.	No aircraft voltage and or engage signal to Yaw module.	Correct power, ground, and/or Engage wiring.
Yaw servo will not move but there is clicking.	No aircraft voltage and or torque signal to Yaw servo.	Correct power & ground. Check @5 voltage at pin 6 at the servo Yaw engaged.
Yaw module on with power only.	The ground on pin 8 at module or wiring error.	Remove the wire at pin 8 and confirm the module does not engage. If yes, call for support. If no, correct wiring from the controller.
Rudder moves in the wrong direction when Yaw Module is tipped.	The module strap or servo wires are incorrect.	Correct wire strap or servo wires per installation instructions.
The rudder does not move when the tail is pushed.	The gyro switch is not in the correct position.	Confirm gyro switch in "ON" position.
The rudder moves in the wrong direction when the tail push test is performed.	Servo wires are incorrect.	Swap servo wires 4 and 5 per installation instructions.
Unable to center ball in flight.	Ground setup centering procedure incorrect or wiring error.	Perform centering procedure per Initial Setup instructions. Confirm pin 4 at module adjust from 0 to 5v when adjusting controller centering.

3.5. The k method, p method, and p-k method for flutter analysis

The k method problem of eigenvalues is still an obstacle, but it predicts accurately the probability of instability when there is a possibility, though distinct of little damping of the structure. The p method predicts the blending of frequency at a minimized velocity (Patil *et al*, 2004).

This equation applied to the software used simulates flight conditions to test for flutter in aircraft. Software like Simcenter 3D and NASTRAN. They make use of the p-k solution to get the true damping results during the tests carried out.

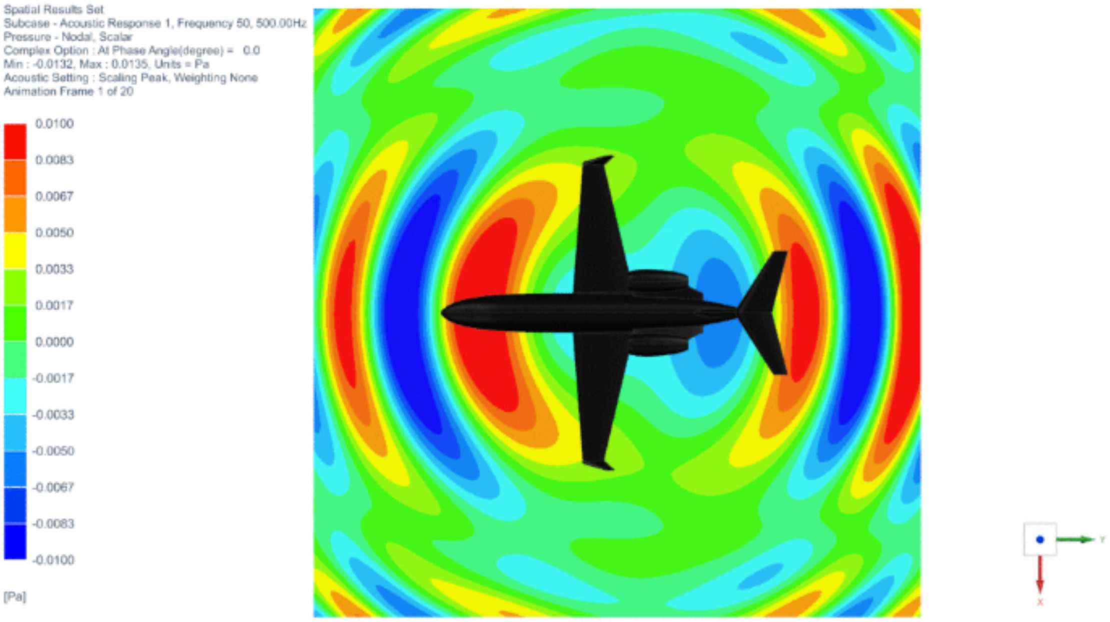

Fig 3.5.0 Simulation of an aircraft using Simcenter3D.

CHAPTER FOUR

RESULTS AND DISCUSSION

4.0 Introduction

The solutions in existence to combat flutter especially the discovery of the new alloy and the damper in the engineering process have led to a significant reduction in the number of aircraft accidents caused by flutter. The interaction between a fixed-wing aircraft and airflow is depicted in Fig 4.0, simulating the flow of air on the surface of the plane, the propeller blades, and especially the wings. At ATA engineering, tests are carried out and simulations operated to determine all the parameters necessary in predicting flutter. The possible adjustments needed to eliminate the probability of early flutter encounters are discovered. The behavior of the aircraft in the same conditions after certain engineering modifications have been made to the aircraft. The necessary corrections have been made to the equations involved in the testing. The software used for these simulations at ATA engineering is Simcenter 3D and or NASTRAN. These two have been pivotal and effective in aircraft design and have allowed for accurate implementation of particular modifications where needed on the aircraft, showing where the issues are and may arise during flight.

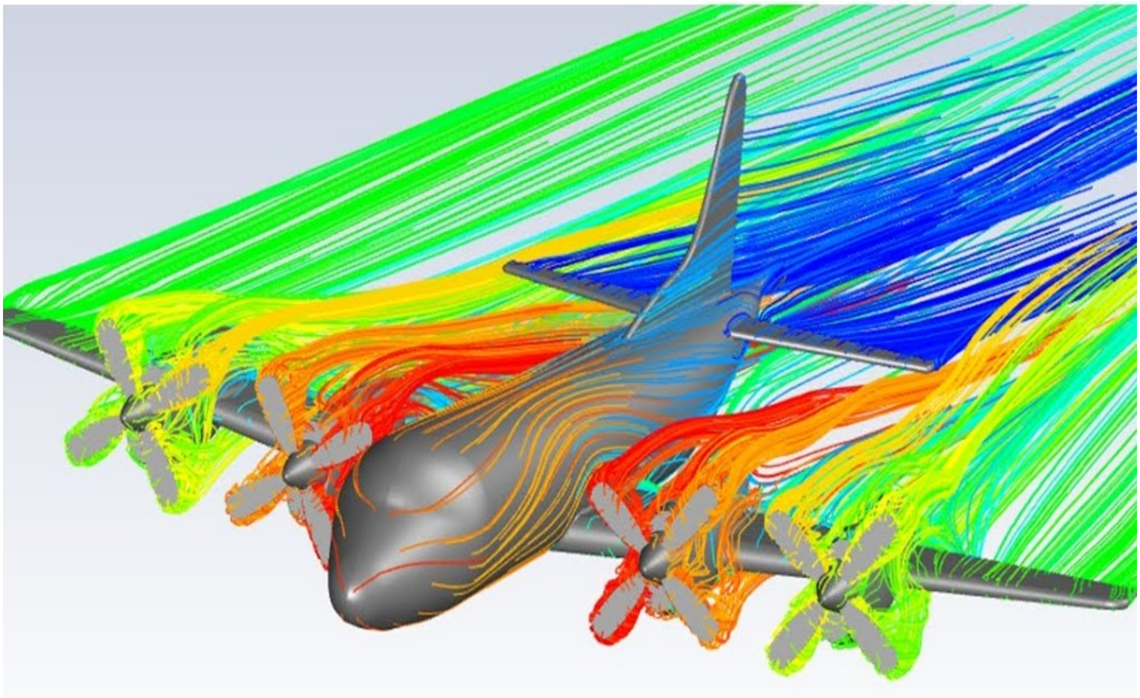

Fig 4.0. Airflow on the surface of an aircraft.

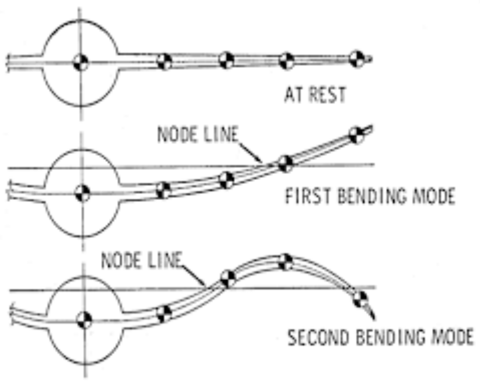

Fig 4.0.1 Wing vibration mode shapes (Weisshaar, 2012).

Damping in a system is a measure to restrain the level of elasticity the system can experience. This is mostly used in structures that are prone to stretching or bending when exposed to forces that displace them from their center of gravity. The phenomenon of flutter is the blending of forces that leads to vibration, that vibration is an oscillatory motion. Let us take an aircraft, for example, flutter is experienced on the wings, the wings vibrate up and down along the z-axis, assuming the wings do not possess any form of damping, it becomes a zero damping motion. This is usually sinusoidal in nature when observed, meaning the flutter is free to act upon the wing as it wishes until the yield strength is surpassed and the wing breaks off.

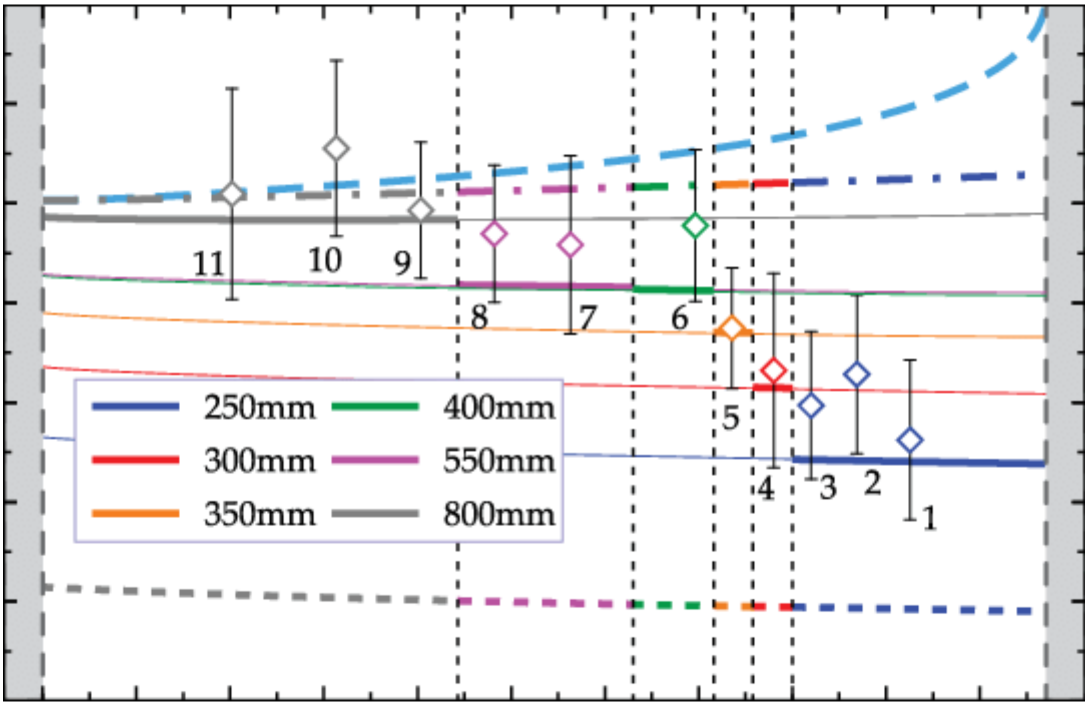

Fig 4.1 Critical flutter load p against mass ratio Theoretical hypotheses based on Eq. (7) are shown by the upper dashed curve, dot-dashed lines, lower dashed lines, and solid lines when both damping processes are active. Diamonds with error bars denote the experimental outcomes. The tested samples are given numbers, and Table I lists their characteristics.(courtesy of Dave Bigoni *et al.*, 2018)

The mass balance applies a level of damping to the wings, reducing the amplitude and wavelength of the vibration. When a mass is applied to an oscillating system the amplitude is reduced and the period minimized. The mass balance changes the position of the center of gravity, this causes the vibration to move to the new center of gravity at a position of higher strength(stiffness) and durability, but the mass balance itself has low strength so it cannot be a permanent solution, though it can still be utilized in aircraft and provide damping to a certain extent, it should not be expected to completely eradicate the chances of flutter.

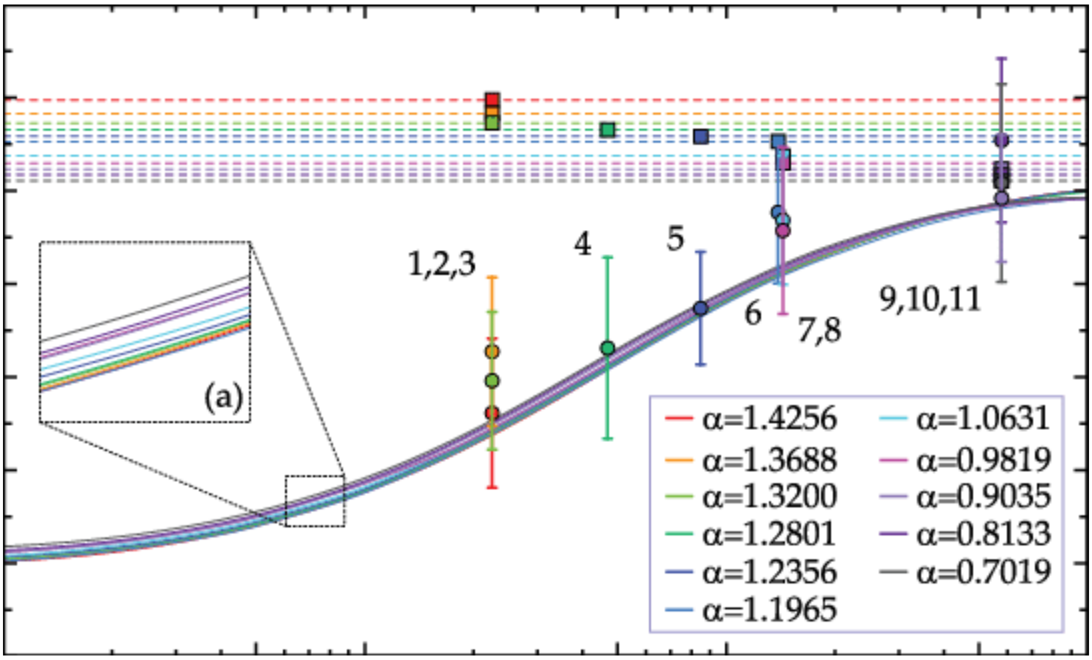

Fig 4.2 See Table I for solid curves that indicate the critical flutter load vs damping ratio at various mass ratio values and related fixed values of. Spots with error bars represent the experimental results. For the same values of, dashed lines represent the essential flutter load of the undamped Pflüger column (courtesy of Davide Bigoni *et al*., 2018)

The flight envelope is a designation of safe and unsafe airspace for an aircraft based on the tests carried out to determine the strength, durability, and maneuverability. Limiting the flight envelope as a solution is not necessarily effective as any airspace no matter the altitude can experience a sudden gust of wind, which is what is responsible for most flutter events. The level of effectiveness is determined by how much understanding is possessed of that region and the pattern observed by the wind, it is still a good method but it is still prey to spontaneous wind activity which makes it not at all reliable.

The aluminum-lithium alloy has proven to be very effective in the fight against flutter. The alloy has a higher strength compared to normal aluminum, it has many variations, each possessing advantages of its own. The wonder of the alloy is how it can increase the stiffness of the wings, resist rust, and remain low-cost, at least lower in price compared to carbon fiber. The only disadvantage is the fine refining process due to the high reactivity of lithium.

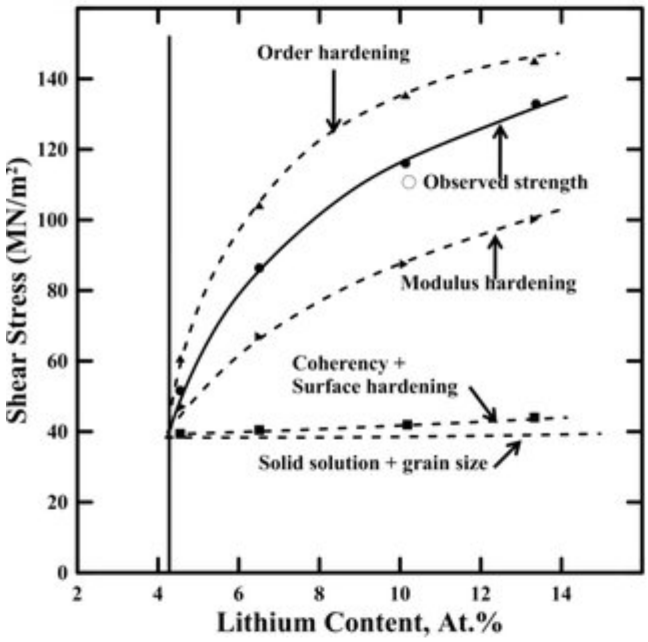

Fig 4.3 Precipitate strengthening in Aluminum Lithium alloy (courtesy of Prasad *et al*., 2017)

The yaw damper is so far the best on-hand method to mitigate flutter, it is very efficient during flights. The parallel and series system of setting up the yaw dampers depends on the design of the aircraft, solutions using graph have been used in auto-control systems in aircraft experiencing saturation effects (Phillips, 1953). The only problem with yaw dampers would be the strength of the actuators and the reaction time. These problems I believe will be solved in the nearest future. Optimization of aircraft design can be split into two classifications, the requirements for functionality, and the requirements for performance (Chakravarty, 1987).

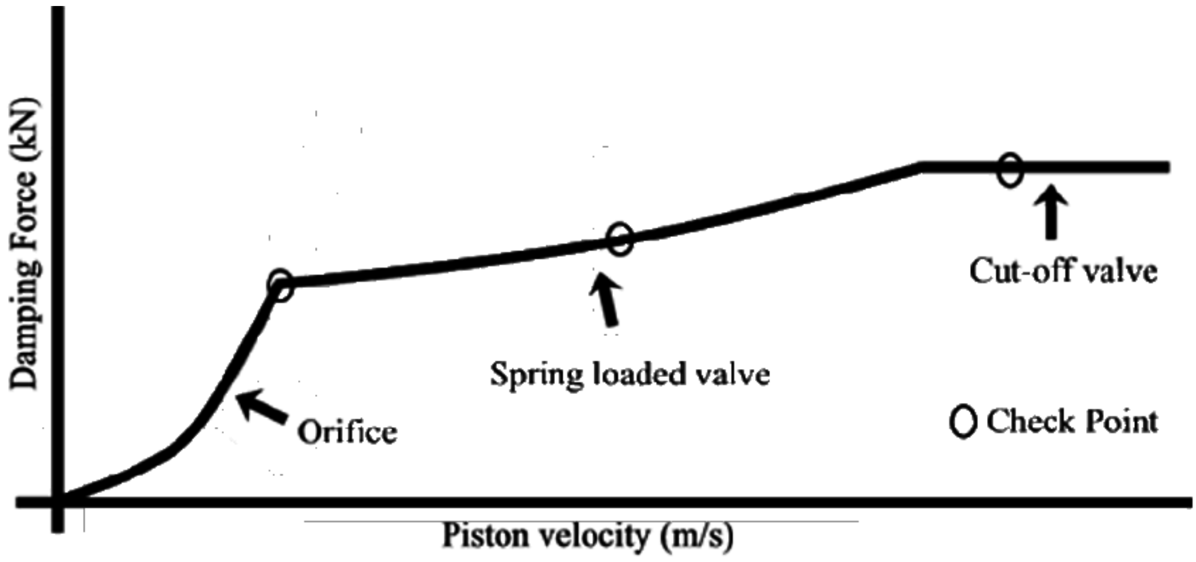

Fig 4.4 Yaw Damper performance (courtesy of Mellado, 2006).

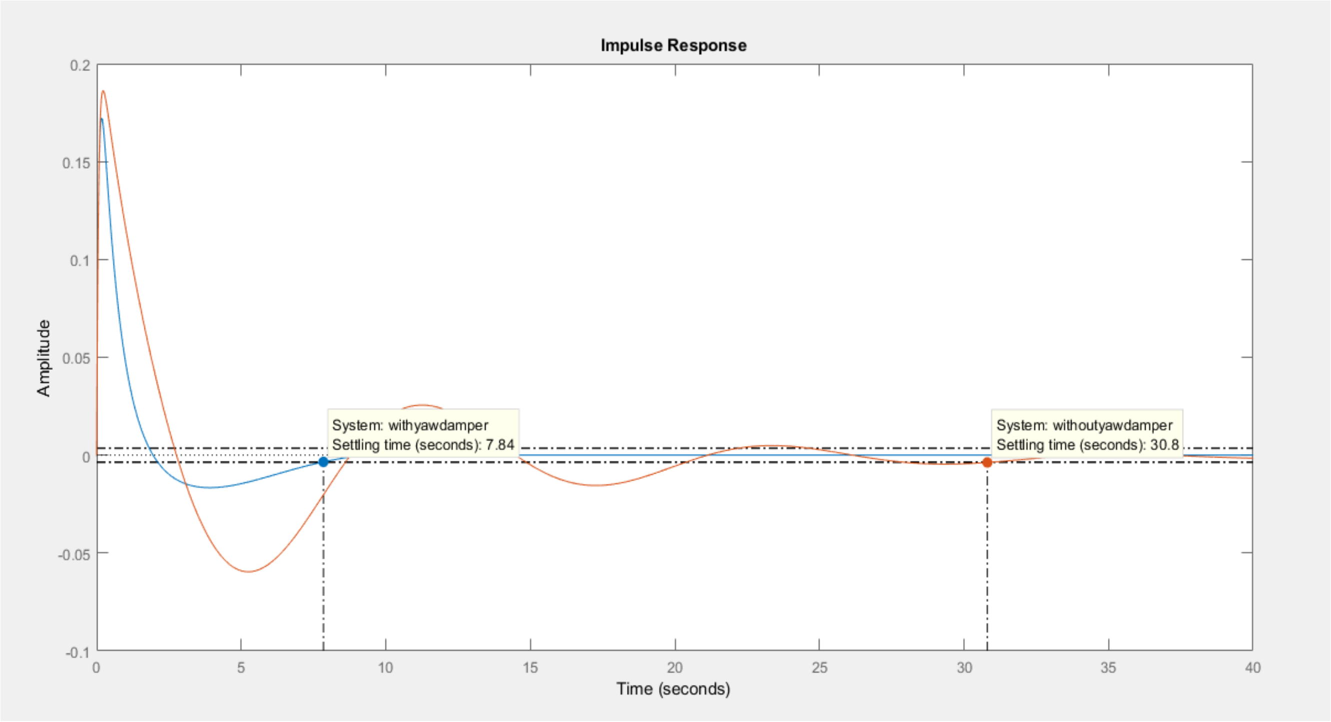

Fig 4.5 Impulse response of a system with and without a yaw damper (courtesy of Kaviyarasu, MIT Aerospace engineering).

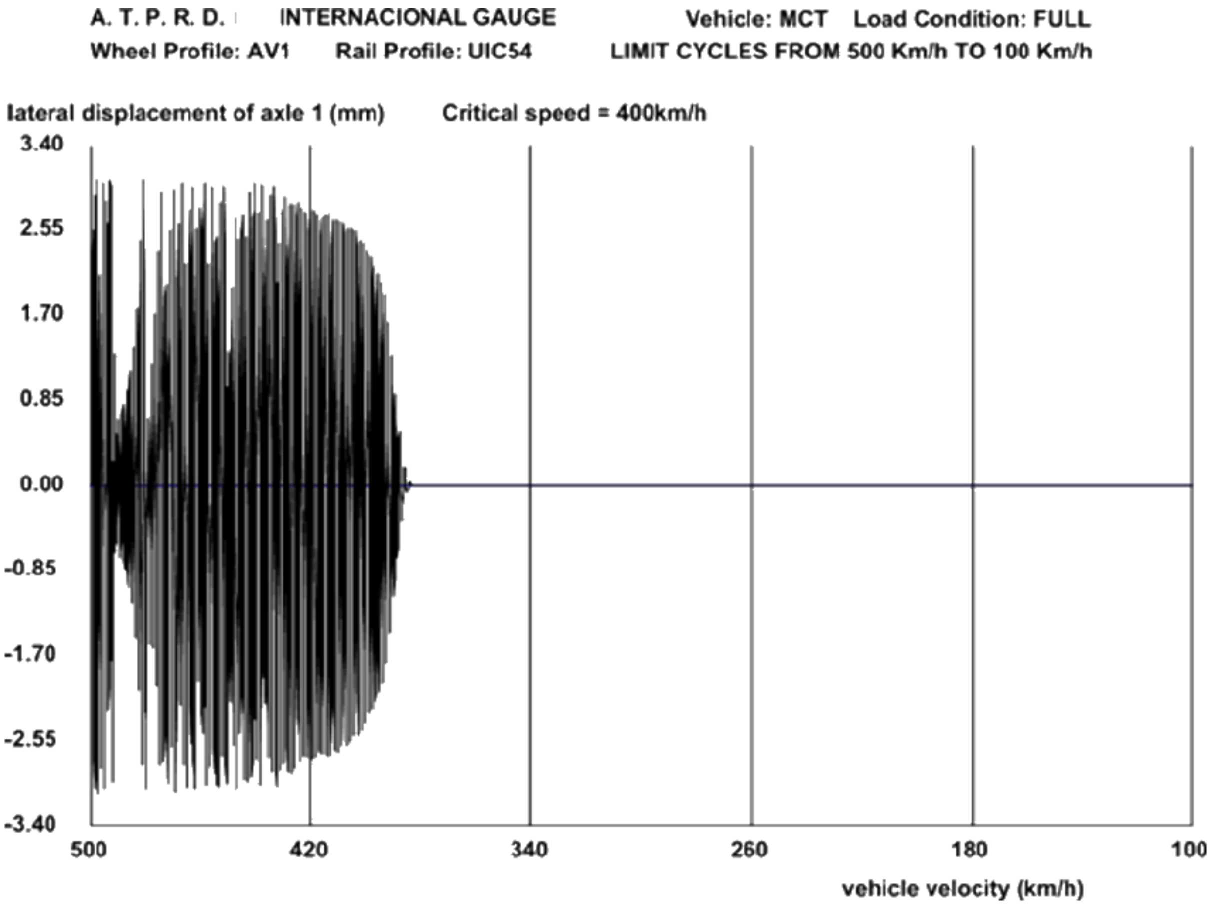

Fig 4.6 limit cycles analysis with corrected yaw damper model (courtesy of Mellado, 2006).

The k-method, p-method, and p-k method for flutter analysis is proving to be quite the trio in the simulating field, each having its advantages. The k-method is always calculated considering the damping on the system to be greater than zero, it does not calculate for a system with zero damping. The p-method is used on systems with zero damping and the p-k method is used the determine the true damping solutions for flutter during tests.

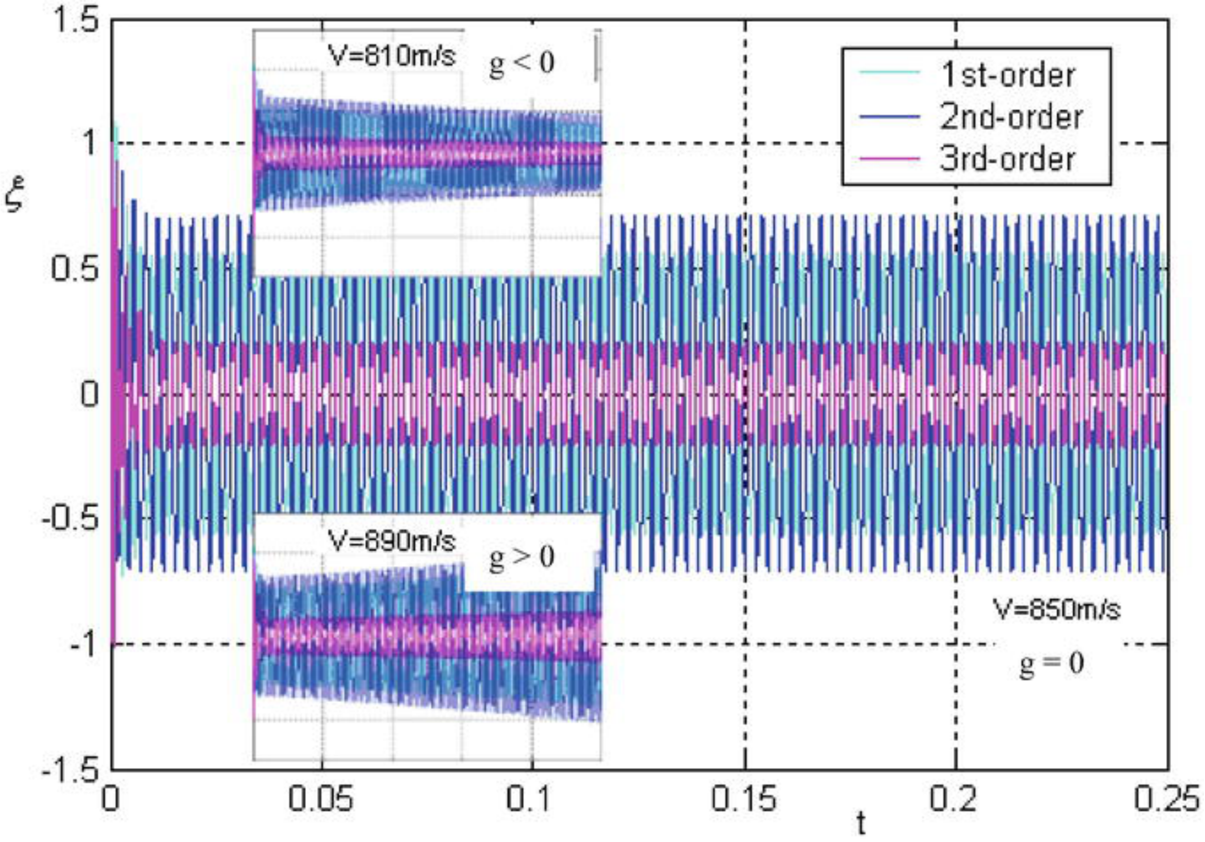

Fig 4.7 The wing flutter calculation, V = 850 m/s. (courtesy of Qiu, 2021).

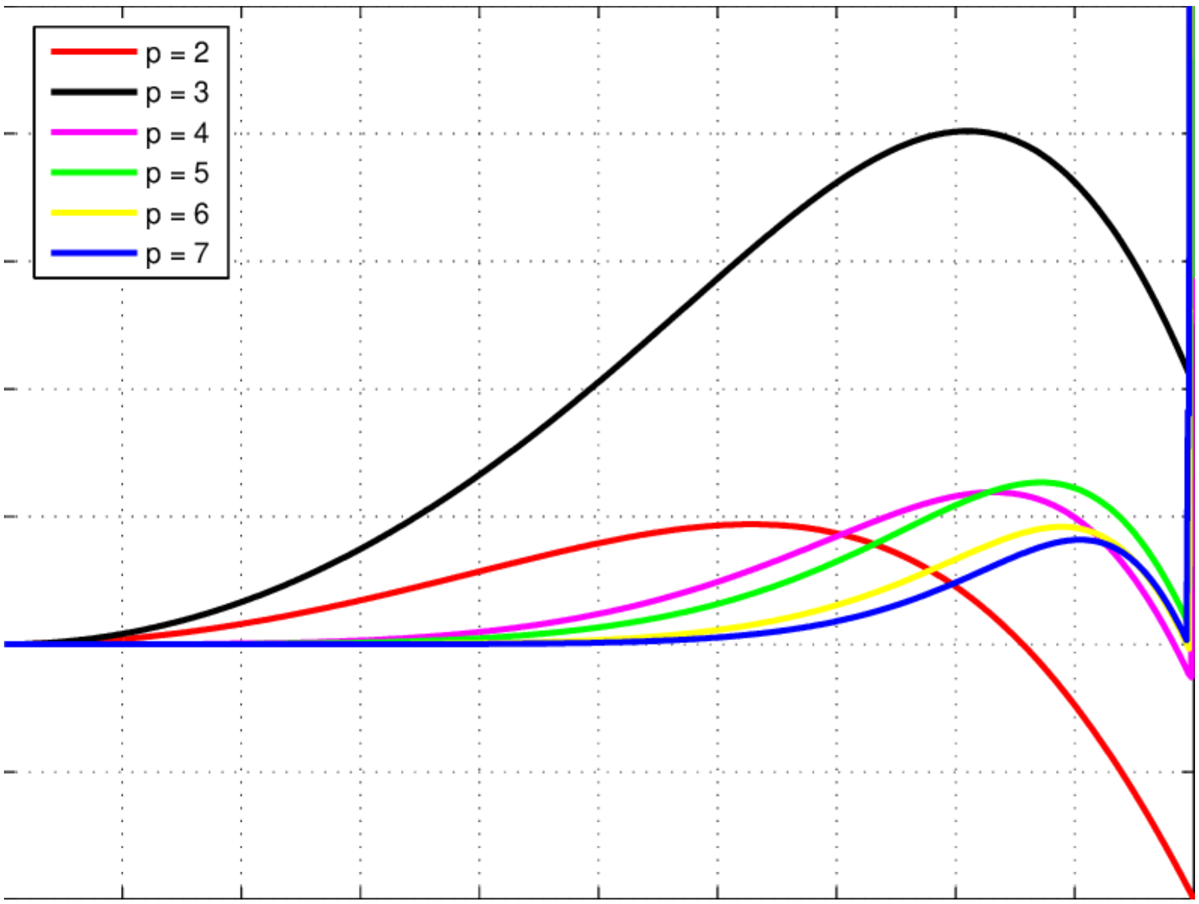

Fig 4.8 Eigenvalue problem in 1D with Dirichlet boundary conditions. Normalized spectra for different degrees of approximation (courtesy of Hughes, 2009).

CHAPTER FIVE

CONCLUSION

5.0 Summary

The progress made over the years in the field of aerodynamics has enabled a firm understanding of the forces involved in the flutter phenomenon. The aerodynamic force, the elastic force, and the inertial force are responsible for the flutter in aircraft, their interaction is still being observed for any new discovery. Before an aircraft is deemed worthy of legal flight, tests are carried out to determine the speed at which the aircraft would reach before flutter occurs (*i.e* flutter speed).

Flutter is a significant issue that, structures interacting with airflow experience, but the solutions provided, for example, the mass balance, yaw damper, aluminum lithium alloy, *e.t.c.* have proven effective to a certain extent. This shows that with more research and experimentation we will be able to reduce its effect to a negligible stage if not eradicate it. The methods available so far have their separate strengths and weaknesses, but with more focus on combining these strengths through experimentation, a competent solution will be achieved.

The several methods mentioned have allowed for a level of control of how much damage is received, but the end goal is to eradicate flutter. Humanity has come far in the battle against this phenomenon, more innovative ideas will be discovered as more pondering is done on the issue.

5.1 Recommendation

The research being done in the alloy-producing section should be further extended, a trio of composite element alloy could be produced to create an effective synergy where each element's inadequacies are canceled out and the strength(stiffness) of the general alloy experiences a significant rise, the over-reactive nature of lithium should be reduced with a non-reactive element. The mathematical model of aircraft should include a variable that accounts for predicting the elasticity of aerodynamic forces.

In addition, the general attitude towards flutter, treating it as just a phenomenon is not the limit. Flutter can be treated as a parameter, in an aircraft there can be a way to objectively state the magnitude of flutter. This would allow for effective observation where simply looking at the build of the aircraft would immediately give an estimate of the flutter tendencies and the degree of flutter in any aircraft.

The idea that flutter occurs at a certain speed should also be viewed with room for more understanding. The forces involved during flutter vary at different speeds, and such variations are not consistent because of the spontaneous nature of natural forces. This gives way for errors we cannot foresee, disrupting our calculations and contradicting our predictions.

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