ANALYSIS OF PHYSICAL VARIABLES FOR MAGNETO-HYDRO DYNAMIC (MHD) FLOW THROUGH A VERTICAL DEFORMABLE LAYER

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DECLARATION

I hereby declare that this project report written under the supervision of DR. S.O. KAREEM is a product of my own research work. Information derived from various sources has been duly acknowledged in the text and a list of references provided. This research project report has not been previously presented anywhere for the award of any degree or certificate.

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Date

CERTIFICATION

This is to certify that the content of this project entitled **“ANALYSIS OF PHYSICAL VARIABLE FOR MAGNETO HYDRO DYNAMICS (MHD) FLOW THROUGH A VERTICAL DEFORMABLE LAYER”** was prepared and submitted by Lateef Abosede Shekinah with matriculation number 17010302002, in partial fulfilment of the requirements for the award of the degree of Bachelor of science in Physics, Department of Physics of the Mountain Top University, Ogun State, Nigeria. The original research work was carried out by her under my supervision and is hereby accepted.

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DR. S.O Kareem

Head of Department

DEDICATION

I dedicate this project to God Almighty for giving me life, good health and all I needed to make this work a success and secondly to my dear parents, Prof. A.O and DR. (Mrs.) O.A Adejumo, for their guidance, understanding, love and sacrifice. I also dedicate this work to my special friend whom words cannot quantify in person of Adekunle Phillip Adedeji and my siblings Nefiu, Ayanfe, Igbega, Amuye, Goodluck, Opeyemi, Eyinju. for their care and always checking up on me, in the course of my four-years of study in Mountain Top University.

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ABSTRACT

This study shows the analysis of physical variables for MHD flow through a vertical deformable layer. The bounding vertical plates y=0 and y= h is sustained at different constant temperature T_0 and T_w respectively. The movement of the fluid and solid deformation are considered in the porous medium. Also, equations governing the velocity of the fluid and displacement of the solid are derived in the vertical deformable porous medium. Substantial results are obtained for the various values of parameters such as Gash of no G_r , magnetic parameters, heat source α, drag coefficient δ, volume fraction of the fluid in porous layer. The Adomian decomposition scheme was also implemented using the software package known as MATHEMATICA. The results obtained from using the MATHEMATICA package were explained using graphical method. The results obtained shows that the drag coefficient increases as the displacement increases and that as the displacement increases the heat source also increases.

CHAPTER ONE

INTRODUCTION

1.0 Background of Study

In physics, viscous flow across a porous media has a number of interesting applications.

The majority of the research focused on flow through rigid (undeformable) porous media.

The theory of deformable porous media comes in handy in these situations to ensure a thorough knowledge of the physical simulations (Terzaghi., 1925), was among the first to investigate fluid flow through deformable porous materials, and (Biot *et al.,* 1962), proposed a model for deformation mechanics and acoustic propagation of fluid flow in porous layers.

In the inclined deformable porous lay er, Gopi Krishna et al. created an entropy production on viscous fluid. (Sreenadh *et al*., 2016), observed free convection flow of a Jeffrey fluid through a vertical deformable porous stratum and MHD free surface flow of a Jeffrey fluid over a deformable porous layer. (Sreenadh *et al*., 2016) looked at viscous fluid flow in a deformable porous media in an inclined channel.

Furthermore, too much energy is squandered or dissipated in the form of heat during the energy producing process. Entropy formation in a couple stress fluid moving through a porous channel with slip at the isothermal walls has been studied by a number of scientists. One of the key parts of fluid dynamics is magnetohydrodynamics (MHD). (Kareem et al., 2016).

1.1 Statement of the Problem

The purpose of this study is to investigate the effects of physical variables and magnetic parameters on a fluid flowing through a deformable layer with a width of **h**. The flow is subjected to some boundary conditions is forced through a porous medium by pressure gradient. therefore, in this study, the use of Adomian decomposition method will be employed to solve the governing equations of the flow.

1.2 Aims and Objective of the Study

The aim of this study is to analyze the effects of physical variables on the fluid flowing through a deformable layer with a width of h.

The objectives of this project are to:

- Rescale the model equations (obtain dimensionless equations).
- Rewrite the equation of motion to conform with Adomian decomposition algorithm.
- Use mathematical package to implement the Adomian decomposition algorithm to obtain the entropy generation rate.

1.3 Significance of the Study

We hope that this research work will be of great help to many industries and thermal engineering processes involving in non-Newtonian fluids.

1.4 Project Outline

Chapter two contains the literature of the problem, where some properties of fluid and the classifications of fluid flow are reviewed. The Adomian decomposition methods are applied to obtain the solution to the governing equations, will be explained in this project. Discussion and the result of the experiment will be presented in chapter four and conclusion and recommendations in chapter five.

CHAPTER TWO

2.0 INTRODUCTION

The main differential equations in fluid mechanics or fluid dynamics will be gotten using a fundamental physical principle in this chapter. Some certain definitions of fluid properties and classifications will be discussed in order to aid our comprehension of some fundamental fluid concepts. The principle of conservation of mass (continuity equation), the principle of conservation of linear momentum (Newtons second law), and the principle of conservation of energy are examples of conservation principles (first law of thermodynamics). In order to apply the above listed principles to fluid mechanics or fluid dynamics, small piece of fluid is typically visualized. this does not alter as the fluid flows from one end to another within its volume. The continuum hypothesis foundation is a particle or parcel, which is a small fraction of the fluid. this chapter also looks at various works done by other researchers and scholars in relation to the research study.

2.1 Fluid

Fluid mechanics is a branch of physics that studies fluid mechanics and forces acting on them. It can be used in a number of different fields. In our daily lives, fluid plays a crucial function. Solid, liquid, and gases are the three basic types of matter. Liquid and gases, on the other hand, appear to have several features that distinguish them from one another.

2.2.0 Fluid Properties

Fluids have a number of physical features that are particularly relevant. Density, pressure, temperature and viscosity are among them. Each of these characteristics will be discussed one after the other.

2.2.1 Density (ρ)

A substance`s density is defined as its mass per unit volume, or the ratio of mass to volume. Other ways to express density include relative density and mass density.

Mass density = $\rho = \frac{mass}{volume}$

Where ρ is the density, m is the mass, v is the volume?

Mass density; Dimension: ML^{-3} unit = kg / m^3

Relative density: SG = $\frac{\rho}{\rho water}$

Relative density: Dimension: $ML^{-2}T^{-2}$ unit = N/m^3.

2.2.2 Pressure (p)

It is a continuous physical force exerted by a fluid per unit area at a perpendicular point.

Pressure;

$$\mathbf{P} = \frac{Force}{Area}$$

Various units are used to express pressure: SI unit in pascal, SI base units: N/m^2, kg/ms^2. Dimension = $ML^{-1}T^{-2}$.

2.2.3 Viscosity

Is a measure of the internal friction of a fluid. It is also a quantity expressing the magnitude of internal friction in a fluid; as measured by the force per unit area resisting uniform flow. This friction becomes apparent when a layer of fluid is made to move in relation to another layer.

$$\mathbf{F} = \mu A\frac{U}{y}$$

Where f is identified as the force, μ is the viscosity of the fluid, A is the area of each plate and $\frac{U}{y}$ is the rate of shear deformation. Other examples of viscosity are described in this study, they include; Kinematic viscosity and Dynamic viscosity.

Kinematic viscosity is the ratio between the dynamic viscosity and mass density. Also, kinematic viscosity is the measure of the internal resistance when the fluid is in motion.

$$V = \frac{\mu}{\rho}$$

The v is known as the kinematic viscosity, and ρ is the mass density, $\mu$is the dynamic viscosity. The kinematic viscosity SI unit is in s/m^2 (square meter per second).

Dynamic viscosity is the tangential force per unit area required to move the fluid in one horizontal plane with respect to another plane.

$$\mu = \frac{\tau}{\gamma}$$

Dynamic viscosity = shear stress / shear rate

The μ is the dynamic viscosity, the τ is known as the shear stress and the γ is the shear rate.

2.2.4 Temperature (T)

Is a system`s degree of hotness and coolness. It can also relate to a substance`s measurement or the transmission of heat energy from one system to another. There are three significant temperature scales: Celsius, Fahrenheit, and Kelvin. Temperature can also be expressed in centigrade (℃) where the freezing point and boiling point of a fluid mostly water is taken from (0℃) and (100℃). The SI unit of temperature is expressed in kelvin.

2.3.0 Classifications of Fluid Flow

2.3.1 Laminar Flow

Laminar flow is categorized by fluid particles or parcel following smooth path in layers, with each layer moving smoothly past the adjacent layers with little or no mixing. At low velocities, the fluid tends to flow without lateral mixing and adjacent layers slide past one another. There are no cross currents perpendicular to the direction of flow, nor eddies or swirls of fluids.

Laminar flow is a flow in which the motion of the particles of fluid is very orderly with particles or parcel close to a solid surface moving in straight line parallel to that surface. Laminar flow is a flow regime characterized by high momentum diffusion and low momentum convection. Also, laminar flow transpires when the fluid flows in infinitesimal parallel layers with no disturbance between them. Turbulence flow is the flow in which the fluid particles move in a disorderly way, the eddies formation takes place which is accountable for energy loss. A process whereby fluid experiences irregular fluctuations, or mixing, in contrast to laminar flow.

2.3.2 Steady Flow

Is a flow in which the amount of fluid flowing per second through any section is constant. Also, it is a flow in which the conditions (velocity, pressure and cross-section) may vary from point to point but do not change with time.

2.3.3 Unsteady Flow

Is a process in which the velocity of a flowing fluid at a specific point change with time. This flow shows temporal increase and decrease in velocity with time which is often related to passage of discrete event.

2.3.4 Compressible and Incompressible Flow

Incompressible flow is a flow with variation of density due to pressure changes is negligible or infinitesimal. All fluid at constant temperature is incompressible. While compressible flow is a flow that experiences a notable variation in density with trending pressure. They are fluid with variable densities.

2.3.5 Continuum Hypothesis

When working with fluids, we tend to overlook the fact that fluids are made up of millions of molecules or atoms randomly moving in a relatively tiny area.

The continuum hypothesis opines that we can associate with any kind of volume of fluid, no matter how small (but should be greater than zero) those macroscopic properties e.g., density, velocity, temperature etc. that we can always associate with the large fluid.

2.3.6 Viscous Flow and Inviscid Flow

Viscous flow is defined as a type of fluid flow in which there is a constant stable motion of the particles, the motion at a fixed point always remains constant. A viscous flow is also called a streamline flow, laminar flow or steady flow. It also opposes the motion of a portion of fluid relative to another. While an inviscid flow is a flow in which the viscosity is the fluid is equal zero.

2.3.7 Conservation of Mass (Continuity Equations)

Conservation of mass states the mass of a system does not alter when moving from one point to another. We can relate the mass of a system to the density of the fluid.

Conservation of mass can be expressed as follows;

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho v) = 0 \qquad \text{............ (2.1)}$$

Or

$$\frac{D\rho}{Dt} + \rho(\nabla \cdot V) = 0 \qquad \text{------------------ (2.2)}$$

These are conservative and non-conservative forms of mass conservation. Where V is the velocity vector, ρ *is the density*, ∇ *is the divergence*, t *is the* time. Conservative forms of equations are derived with the applications of physical principles to a fluid element in a given space in which non-conservative forms can be produced by investigating fluid elements passing through the flow field.

$$\frac{D\rho}{Dt} = \frac{\partial \rho}{\partial t} + \rho(\nabla \cdot V) \qquad \text{............... (2.3)}$$

The relationship between the two equations can expressed using the following general equation that connect spatial and material descriptions of fluid flow. Therefore, let A be the material derivative or acceleration vectors

$$\frac{DA}{Dt} = \frac{\partial A}{\partial t} + \mathrm{A}(\nabla\ .\mathrm{V}) \qquad \text{………… (2.4)}$$

The material derivative of the property (the moving element in the flow fluid) at the L.H.S of the equation, whereas the partial time derivative appears at the R.H.S (the fluid element in a foxed space) the conservative derivation of A is the final term.

2.3.8 Conservation of Linear Momentum

Conservation of momentum states that the time rate of change of momentum of a system of particles is equivalent to the sum of external forces acting on that body. Conservation of linear momentum is also recognized as Navier stokes equation.

$$\frac{d}{dt}\int \rho u dv = \int \rho g dv + \int T ds \qquad \text{…………………………….. 2.5}$$

Where g is the body force per unit mass and T is the surface force per unit surface area bounding V. but if the volume is small enough, the integrands can be taken out of the integral.

$$\frac{d}{dt}\int \rho v dv = \frac{d}{dt}$$

CHAPTER THREE

METHODOLOGY

3.0 INTRODUCTION

This chapter describes the model and the methodology used in this project. the method of solution for Magnetohydrodynamic (MHD) fluid flow. The equations obtained from the model are usually dimensional. In order to reverse the dimensionality in the equations, the equations are rescaled into dimensionless form. After changing the dimensional equations to dimensionless forms, we applied the Adomian decomposition method to obtain the recursive schemes that were solved using the mathematical symbolic package.

3.1 The Formulation of the Mathematical Model

In this project, we considered a channel with two boundary plates. The channel is a vertical deformable porous channel that allows the flow of fluid vertically. In this channel, the plates are parallel to the vertical axis which is labeled x-axis. The horizontal axis is perpendicular to the vertical axis and is labeled y- axis. The width of the channel is **h**. Also, there is a force that pushes up the fluid along the porous medium and the force is produced by the pressure gradient $\frac{\partial p}{\partial x}$ and with the application of magnetic field of strength B_0 perpendicularly to the plate. Fig 1 shows the geometry of the model. According to Sreenadh et al (2018), three dimensional equations are necessary to describe the model. They are the momentum balance equation, the energy balance equation and the local entropy generation rate equation. The three equations were re-written in dimensionless forms, starting from the first as follows:

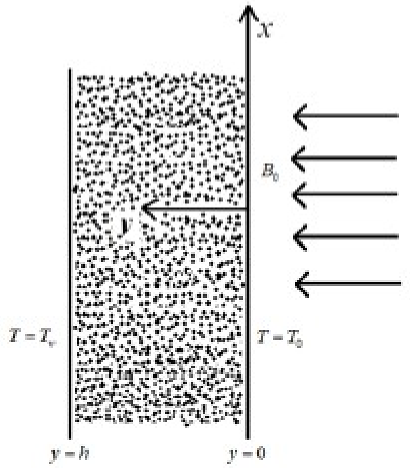


FIG. 1: Physical model

3.2 Dimensionless Forms of the Equations

The momentum balance equation is given as;

$$\mu \frac{d^2 u'}{\partial y'^2} \; -(1\text{-}\varphi)\, \frac{\partial p'}{\partial x'} + ¿\ K v' = 0 \qquad 3.1$$

Where μ is lame constant, K is drag coefficient, φ is volume fraction of the fluid and x and y are the dimensionless cartesian coordinates and(x' y') are the cartesian coordinate.

Let u = $\frac{u' \mu}{\mu_f U}$,y = $\frac{y'}{h}$, v = $\frac{y'}{U}$

3.2

Where μ_f is the coefficient of viscosity, U is the average velocity of the fluid in the porous channel, v is the velocity at which the fluid moves in the channel, u is the solid displacement and h is the width of the channel.

$$\frac{d}{d y'} = \frac{d}{dy} \times \frac{d}{d y'}$$

$$\frac{d}{dy} \times \frac{1}{h} = \frac{1}{h}\frac{d}{dy} \qquad 3$$

This implies that:

$$\frac{d^2}{d y'^2} = \frac{1}{h^2}\frac{d^2}{d y^2}$$

$$\mu[\frac{1}{h^2}\frac{d^2}{d y^2}(\frac{uU\mu_f}{\mu})] - (1 - \varphi)\frac{\partial p'}{d x'} + ¿ \text{ KvU} = 0 \qquad 3.4$$

$$\frac{U\mu_f d^2 u}{h^2 d y^2} - (1-\varphi ¿ \frac{\partial p'}{\partial x'} \text{ KvU} = 0$$

$$\frac{h^2}{U\mu_f}[\frac{U\mu_f}{h^2}\frac{d^2 u}{d y^2}] - \frac{h^2}{U\mu_f}[(1-\varphi)\frac{\partial p'}{\partial x'}] + ¿ \frac{h^2}{U\mu_f}[\text{ KvU}] = 0 \qquad 3.5$$

$$\frac{d^2 u}{d y^2} - \frac{h^2}{U\mu_f}(1-\varphi)\frac{\partial p'}{\partial x'}\frac{+h^2 KvU}{U\mu_f} = 0$$

$$\frac{d^2 u}{d y^2} - \frac{h^2}{U\mu_f}(1 - \varphi ¿ \frac{\partial p'}{\partial x'} + ¿ \frac{h^2 Kv}{\mu_f} = 0 \qquad 3.6$$

We rewrite the above equation as;

$$\frac{d^2 u}{d y^2} - (1-\varphi ¿ \frac{\partial}{\partial x'}(\frac{h^2 p^2}{U\mu_f} ¿ + \frac{h^2 KV}{\mu_f} = 0 \qquad 3.7$$

We then define the dimensionless x quantity as;

$$\text{X} = \frac{x'}{h} \qquad \text{then, } \frac{dx}{d x'} ¿ \frac{1}{h}$$

But $\frac{d}{d x'} = \frac{d}{dx} \times \frac{dx}{d x'}$

$$\frac{d}{d x'} = \frac{1}{h}\frac{d}{dx}$$

Using this in equation (3.7) we have

$$\frac{d^2 u}{d y^2} - (1-\varphi ¿ ¿ \frac{d}{dx}\left(\frac{h^2 p'}{U\mu_f}\right) ¿ + \frac{h^2 KV}{\mu_f} = 0$$

$$\frac{d^2 u}{d y^2} - (1-\varphi ¿ \frac{d}{dx}\left(\frac{h p'}{U\mu_f}\right) + \frac{h^2 KV}{\mu_f} = 0 \qquad 3.8$$

Let p = $\frac{h p^{'}}{U \mu_f}$, where p is the dimensionless pressure.

$$\frac{d^2 u}{d y^2} - (1\text{-}\ \varphi ¿ \frac{dp}{dx} + \delta v = 0$$

Where δ = $\frac{h^2 K}{\mu_f}$. we again rewrite the above equation as;

$$\frac{d^2 u}{d y^2} - (1\text{-}\ \varphi ¿ P + \delta v = 0, \qquad 3.8$$

where P = $\frac{dp}{dx}$.

The next to be written in dimensionless form is the energy balance equation. The energy balance equation is given as;

$$2\mu_a \frac{\partial^2 v}{\partial y'^2} - \varphi \frac{\partial p}{\partial x} - K v^{'} - \sigma B_0 v^{'} + g\rho\beta(T\text{-}T_0) = 0 \qquad 3.9$$

Where μ_a is the apparent viscosity of the fluid in porous material, ρ is density of the fluid, g is acceleration due to gravity, B_0 is the magnetic field strength perpendicular to the plate, $K_0$is thermal conductivity, T is the field temperature, T_w is the wall temperature at y = h , T_0 is the wall temperature at y = 0, P is the pressure gradient, M is the magnetic parameter, Q_0 is the heat source, G_r is the grashof number, δ is viscous drag, θ is the temperature and η is the ratio of bulk fluid viscosity to apparent fluid viscosity in porous layer.

Then, we define some basic variables as follows;

Let v = $\frac{v^{'}}{U}$

Which is given as $v^{'}$= Uv

Y = $\frac{y^{'}}{h}$ and this become

$$y^{'} = hy \qquad 3.10$$

$$\frac{d}{dy'} = \frac{d}{dy} \times \frac{dy}{dy'} \qquad 3.11$$

From equation (3.11),

$$\frac{d}{dy'} = \frac{d}{dy} \times \frac{1}{h}$$

Which can be written as;

$$\frac{d}{dy'} = \frac{1}{h}\,\frac{d}{dy} \qquad 3.12$$

Then, we substitute for the dimensionless variables and differentiate the coefficients.

$$\frac{2\mu_{aU}}{h^2}\,\frac{d^2 v}{dy^2} - \frac{\varphi}{h}\,\frac{dp'}{dX} - \text{Kuv} - \sigma B_0 \text{Uv} + g\rho\beta\,\text{¿} = 0$$

$$\frac{d^2 v}{dy^2} - \varphi\frac{d}{dx}\left(\frac{hp'}{2\mu_a U}\right) - \frac{h^2 Kv}{2\mu_a} - \frac{h^2 \sigma B_0 v}{2\mu_a} + \frac{h^2 g\rho\beta(T_w - T_O)\theta}{2\mu_a U} = 0. \qquad 3.13$$

Let $\mu_f = 2\mu_a$, P $= \frac{hp^1}{\mu_f U}$

This can also be written as

$$\frac{d^2 V}{dy^2} - \varphi\frac{dp}{dx} - \frac{h^2 K}{\mu_f}V - \frac{h^2\theta\beta_0^2 V}{\mu_f} + h^2 g\rho\beta\,\text{¿¿} = 0 \qquad 3.14$$

Let $\delta = \frac{h^2 K}{\mu_f}$, m $= \frac{h^2\sigma\beta_{0V}^2}{\mu_f}$ + $G_r = \frac{g\beta(T_w - T_0)Ph^2}{\mu_f U}$

This can also be written as;

$$\frac{d^2 V}{dy^2} - \varphi\frac{dp}{dx} - \delta V - MV + G_r\theta = 0 \qquad 3.15$$

This can also be written as;

$$\frac{d^2 V}{dy^2} - \varphi P - (\delta + M)V + G_r\theta = 0, \qquad 3.16$$

where P=$\frac{dp}{dx}$

Re-writing Eqn (3.16),

$$\frac{d^2V}{dy^2}-(\delta+M)V-\varphi P+G_r\theta=0 \qquad 3.17$$

For η= $\frac{\mu_f}{2\mu_a}$,

We can write

$$\frac{d^2V}{dy^2}-(\delta+M)\eta V-(\varphi\eta)P+\eta G_r\theta=0. \qquad 3.18$$

Also, we write the local entropy generation rate equation as follows

$$K_0\frac{\partial^2 T}{\partial y'^2}+Q_0=0 \qquad 3.19$$

We also define some basic variables here which are;

$\theta=\frac{T-T_0}{T_{w-T_0}}$, this can also be written as

T- T_0 = $(T_w - T_0 ¿ \theta$

T = $(T_{w-¿¿}\, T_0\,)\theta+T_0$

Then,   y = $\frac{y'}{h}$

Which can also be written as;

$\frac{d}{dy'}=\frac{1}{h}\frac{d}{dy}$ this is equal to

$$\frac{d^2}{dy'}=\frac{1}{h^2}\frac{d^2}{dy^2} \qquad 3.20$$

Substitute Eqn (3.20) into (3.19) we have;

$$\frac{K_0}{h^2}\frac{d^2}{dy^2}¿- T_0)\theta+T_0\,] + Q_0 = 0 \qquad 3.21$$

Re writing Eqn (3.21) we have;

$$\frac{K_0}{h^2}¿-T_0)\theta+\frac{d^2}{dy^2}T_0] + Q_0 = 0 \qquad 3.22$$

$$\frac{K_0}{h^2}(T_w - T_0)\frac{d^2}{dy^2}\theta+Q_0 = 0 \qquad 3.23$$

We can also re write Eqn (3.23) as;

$$\frac{d^2\theta}{dy^2} + \frac{h^2 Q_0}{K_{0¿¿}¿} = 0 \qquad 3.24$$

Let $\alpha = \frac{h^2 Q_0}{K_{0¿¿}¿} = 0$

Then we have;

$$\frac{d^2\theta}{dy^2} +\alpha=0 \qquad 3.25$$

Equations (3.8), (3.18) and (3.25) are the dimensionless form of the equation

$$\frac{d^2u}{dy^2} -(1-\varphi)\,p+\delta v=0 \qquad 3.8$$

$$\frac{d^2v}{dy^2}-(\delta+M)\eta v-¿\eta)\ p + \eta G_r\theta=0 \qquad 3.18$$

$$\frac{d^2\theta}{dy^2}+\beta=0 \qquad 3.25$$

The three equations (3.8, 3.18, 3.25) are the dimensionless forms of equations (3.1, 3.9, 3.19) respectively.

The boundary conditions are;

At y = 0: u = 0, v = 0, θ = 0

At y = 1: u = 0, v = 0, θ= 1.

3.2 Adomian Decomposition Method. (ADM)

in this section, the three dimensionless equations, that is, eqns (3.8, 3.18 and 3.25) were cast in recursive scheme using the Adomian decomposition method (ADM). The following are the procedure taken to achieve the recursive schemes for the dimensionless equations.

3.3 Recursive scheme for solid displacement.

$$\frac{d^2u(y)}{d y^2}-(1-\varphi)p+\delta v(y)=0 \qquad 3.8$$

Making the term with the highest order of differential coefficient the subject of the equation,

$$\frac{d^2u(y)}{d y^2}=(1-\varphi)p-\delta v(y)$$

The adomian decomposition method (ADM) is a direct method. It does not involve intermediate processes such as linearization or perfusation, which may modify or change the behavior of the equation. The method decomposes the unknown function in eqn (3.8) giving a series of terms. This series is written as;

$$u(y) = \sum_{n=0}^{\infty} u_n(y); n\geq 0. \qquad 3.26$$

Equivalently,

$u(y) = u_0+u_1+u_2+\ldots\ldots\ldots+u_\infty$

The terms in the series, that is, $u_0, u_{1,} u_{2,}\ldots\ldots$ are obtained recursively. This means $u_0(y)$ is needed to obtain $u_1(y)$ and the latter is needed to obtain $u_2(y)$, etc. To obtain the recursive scheme, we rewrite eqn (3.8) in operator forms.

Therefore,

$$Lu(y) = (1-\varphi ¿ P-\delta v(y) \qquad 3.27$$

Where L = $\frac{d^2}{d\,y^2}$

The differential operator $\frac{d^2}{d\,y^2}$ is invertible. The inverse of L therefore, is the integral operator, $\iint(.)\,dydy$ or L^{-1}.

Applying L^{-1} to eqn (3.27), we have;

L^{-1}[Lu(y)] = $L^{-1}[(1-\varphi)P-\delta v(y)]$ 3.28

Let Lu(y) = (.)

Therefore,

$$\frac{d^2}{d\,y^2}u(y)=(.)$$

Then we have;

$$\frac{d}{dy}u(y)=\frac{d}{dy}u(y)\text{¿}_{y=0}+\int_0^y(.)\,dy$$

U(y) = u(y)$\text{¿}_{y=0}+\int_0^y\frac{du(y)}{dy}dy\,\text{¿}_{y=0}+\iint_0^y(.)\,dydy$

Therefore,

$$\iint_0^y(.)\,dydy=u(y)-u(0)-\int_0^y\frac{du(0)}{dy}dy \qquad 3.29$$

Substituting eqn(3.29) into eqn(3.28) we have;

U(y) – u (0) - $\int_0^y\frac{du(0)}{dy}dy=L^{-1}[(1-\varphi)P-\delta V(y)]$

From the boundary conditions, u (0) = 0.

This implies that;

U(y) = $\int_0^y \frac{du(0)}{dy} dy + L^{-1}[(1-\varphi)P - \delta v(y)]$ 3.30

Let $\frac{du(0)}{dy} = ¿$ f

Therefore, we have;

$$\int_0^y \frac{du(0)}{dy} dy = \int_0^y f dy = fy$$

Rewriting eqn (3.30), we have;

U(y) = fy + $L^{-1}[(1-\varphi)P - \delta v(y)]$ 3.31

The expected series solutions for u(y) and v(y) are;

U(y) = $\sum_{n=0}^{\infty} u_n(y)$, v(y) = $\sum_{n=0}^{\infty} u_n(y)$ 3.32

Putting eqn (3.32) in eqn (3.31) we have;

$$\sum_0^{\infty} u_n(y) = fy + L^{-1}[(1-\varphi)P - \delta \sum_0^{\infty} v_n(y)]$$

$u_0 + u_1 + u_2 + \ldots = fy + L^{-1}(1-\varphi)P - \delta L^{-1} ¿$ ……..]

For the recursive scheme, we have:

$u_0(y) = fy + L^{-1} ¿$) P 3.33

Which is equal to;

Fy + $\int_0^y ¿¿$

Fy + $\int_0^y (1-\varphi) Py\, dy$

Fy + $\frac{1}{2}(1-\varphi) P y^2$

Where;

$U_{k+1}(y) = -\delta L^{-1} v_k(y); k \geq 0$ 3.34

Clearly, for the recursive solution by (ADM)

$u_0(y) = fy + \frac{1}{2}(1 - \varphi ¿ P y^2$

$u_1(y) = -\delta \int_0^y \int_0^y v_0(y)\, dydy$

$u_2(y) = -\delta \int_0^y \int_0^y v_1(y)\, dydy$

$u_3(y) = -\delta \int_0^y \int_0^y v_2(y)\, dydy$.

U(y) = $u_0 + u_1 + u_2 + u_3 + ¿$……. 3.35

Eqn(3.33) and eqn(3.34) gives the recursive scheme for u(y). for v(y).

Then, we solve for the second dimensionless equation i.e eqn(3.18) using ADM.

$\frac{d^2 v(y)}{d y^2} - (\delta + M)\eta v - \varphi \eta P + ¿\eta G_r \theta = 0$ 3.18

Like before, we make the term with highest order of differential coefficient the subject of the equation. This gives

$\frac{d^2 v(y)}{d y^2} = (\delta + M)\eta v + \varphi \eta P - \eta G_r \theta$ 3.36

Writing eqn(3.36) in operator form we have;

Lv(y) = $¿)\eta v + \varphi \eta P - \eta G_r \theta$

Multiplying both sides by L^{-1},

$L^{-1} Lv(y) = L^{-1} ¿\eta v + \varphi \eta P - \eta G_{r\theta}]$ 3.37

Let L(v) = (.)

$$\frac{d^2 v(y)}{dy} = \frac{dv(y)}{dy} ¿_{y=0} + \int_0^y (.)dy$$

V(y) = v(y)$¿_{y=0} + \int_0^y \frac{dv(y)}{dy} ¿_{y=0} + \int_0^y \int_0^y (.)dydy$

Using this in eqn(3.37),

V(y) = v (0) + $\int_0^y jdy + L^{-1} ¿¿ \delta + M ¿ \eta v + \varphi$ηP $- ¿$η$G_r \theta ¿$

Where j is equal to

$\frac{dv(y)}{dy} ¿_{y=0}$

This means;

V(y) = jy + $L^{-1}(\delta + M)\eta v + L^{-1} ¿$ηP)$- L^{-1}(\eta G_r \theta)$

Therefore;

V(y) = jy + $L^{-1} ¿$ηP) $+ L^{-1} ¿$ηv - η$G_r \theta$

To obtain the recursive scheme for v(y),

$v_0(y) = jy + L^{-1} ¿$ηP)

This implies that;

$$v_0(y) = jy + \int_0^y \int_0^y \varphi \eta P y dy$$

Gives:

jy$+ \int_0^y \varphi \eta P y dy$

therefore, we have:

$v_0(y) = jy + \frac{1}{2}\varphi\eta P y^2$ 3.38

$v_{k+1}(y) = L^{-1}¿ + M ¿\eta v_k - ¿\eta G_r \theta_k ¿; k \geq 0$ 3.39

More clearly,

$v_0 = jy + \frac{1}{2}\varphi\eta P y^2$

$v_1 = L^{-1}¿\eta v_0 - ¿\eta G_r \theta_1 ¿$

$v_2 = L^{-1}¿\eta v_1 - \eta G_r \theta_1$ etc.

V(y) = $v_0 + v_1 + v_2 + ¿$ …………

For the energy balance equation,

$\frac{d^2\theta}{d y^2} + \beta = 0$ 3.25

$\frac{d^2\theta}{d y^2} = \beta$ 3.40

$\theta(y) = \sum_{n=0}^{\infty} \theta_n(y); n \geq 0$ 3.41

Then,

L$\theta(y) = \beta$

Multiply both sides by L^{-1}

$L^{-1} L\theta(y) = -L^{-1}(\beta)$ 3.42

Let L ($\theta ¿ = (.)$

Then,

$$\frac{d^2\theta(y)}{d\,y^2}=(.)$$

$$\frac{d\theta(y)}{dy}=\frac{d\theta(y)}{dy}¿_{y=0}+\int_0^y(.)dy$$

$$\theta(y)=\theta(y)¿_{y=0}+\int_0^y\frac{d\theta(y)}{dy}¿_{y=0}+\int_0^y\int_0^y(.)dydy$$

$$\int_0^y\int_0^y(.)dydy=\theta(y)-\theta(0)-\int_0^y\frac{d\theta(0)}{dy}dy \qquad 3.43$$

Substitute eqn(3.42) into(3.41)

$$\theta(y)-\theta(y)¿_{y=0}-\int_0^y\frac{d\theta(y)}{dy}¿_{y=0}dy=-L^{-1}\beta$$

$$\theta(y)=\theta(0)+\int_0^y\frac{d\theta(y)}{dy}dy-L^{-1}\beta$$

From the boundary conditions,

When y = 0, $\theta=0$

Also, let $\frac{d\theta(0)}{dy}=q$

This implies

$$\int_0^y\frac{d\theta(0)}{dy}dy=\int_0^y qdy=qy$$

Therefore,

$$\theta(y)=qy-L^{-1}\beta$$

$$qy-\int_0^y\int_0^y\beta dydy$$

$$qy - \frac{1}{2}\beta y^2$$

$$\theta(y) = \sum_0^\infty \theta_n(y)$$

$$\theta_0(y) = qy - \frac{1}{2}\beta y^2 \qquad 3.44$$

Therefore,

$$\theta_1(y) = \theta_0(y)$$

In conclusion, the recursive scheme for the dimensionless model equations were obtained in Eqns (3.33-3.34), (3. 38-3.39) and (3.41-3.44). We have used the mathematical symbolic package to simplify the recursive equation, and to output the results. The results were displayed in graphical forms and discussed in chapter four.

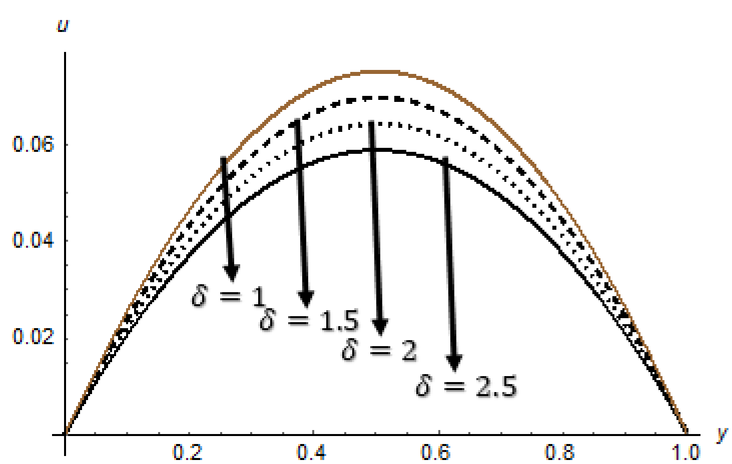
CHAPTER FOUR

RESULTS AND DISCUSSION

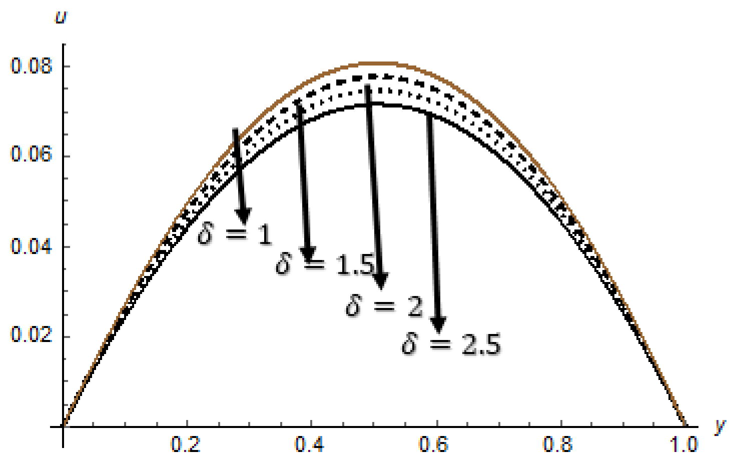
4.0 INTRODUCTION

Natural convection effects on conducting steady flow through a vertical deformable porous layer were investigated in this study. Analytically, the fluid velocity, solid displacement, and temperature distribution were determined. The properties of various physical parameters, such as the volume fraction of the fluidφ , Grashof number, G_r, drag δ, Magnetic parameter M, heat sourceα and the ratio of the bulk fluid viscosity η on the solid displacement, fluid velocity, temperature and entropy generation were solved and discussed graphically in this chapter using software package called MATHEMATICA.

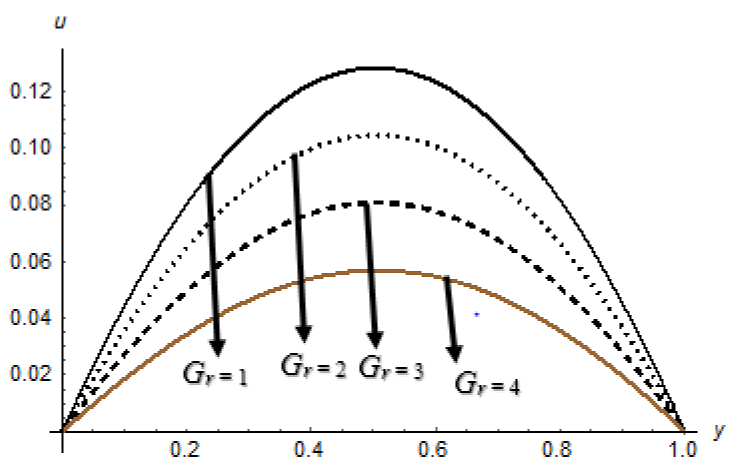
The following graphs representing the displacement profile of different parameters which includes Viscous drag, heat source, G_r Grashof number, volume fraction of fluid, magnetic parameters and temperature.



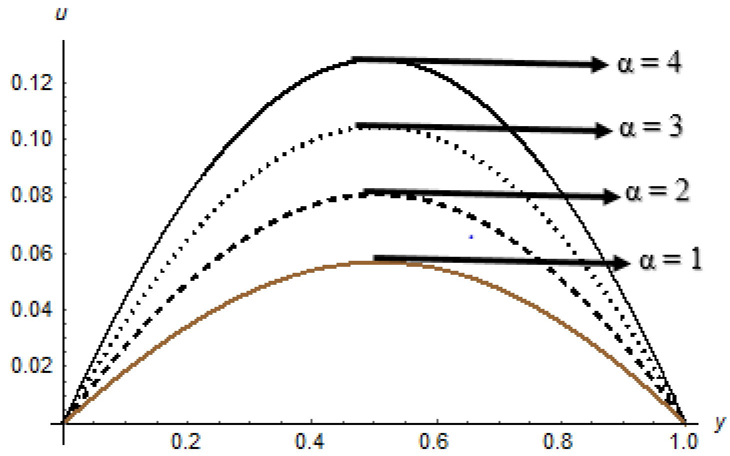
4.1 Displacement profile with respect to viscous drag(δ)



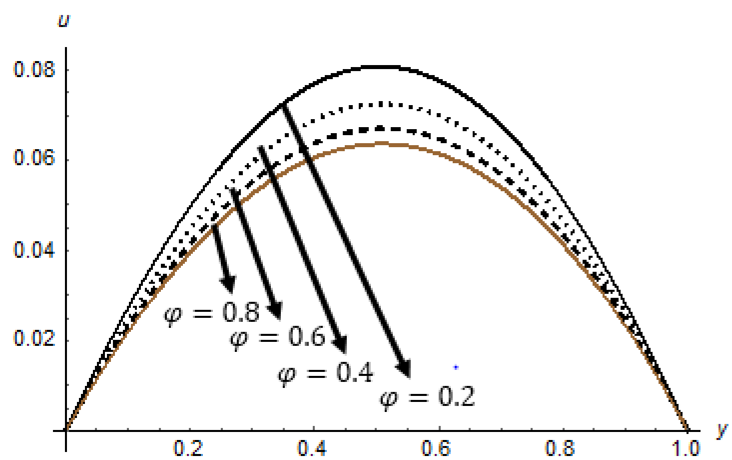
4.2 Displacement profile with respect to δ(viscous drag).



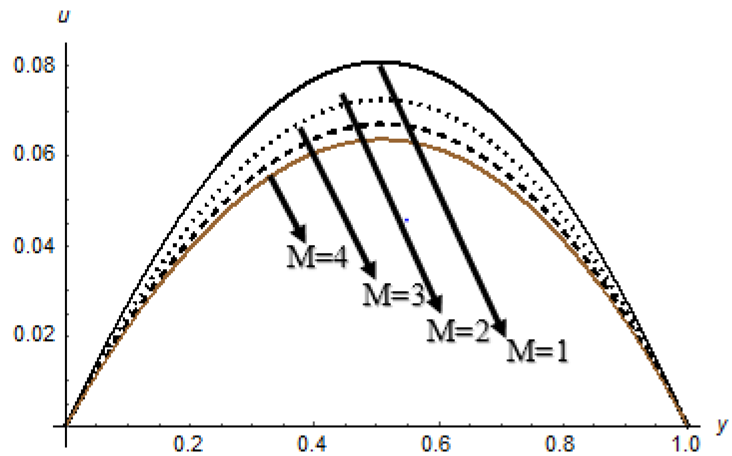
4.3 Displacement profile with respect to G_r(Grashof number).



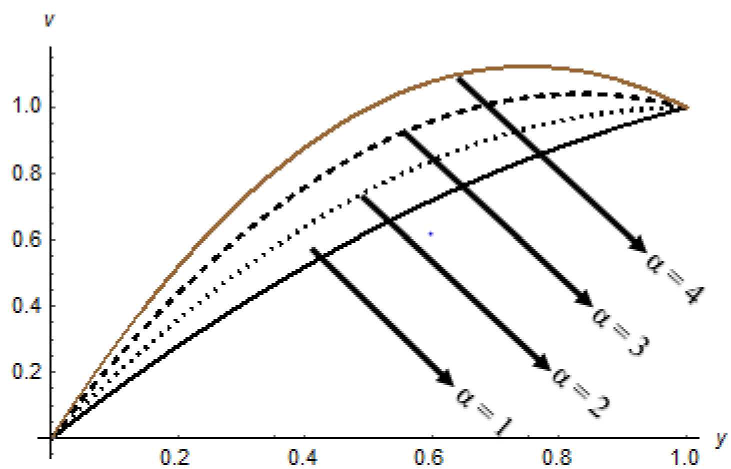
4.4 Displacement profile with respect to α (heat source).



4.5 Displacement profile with respect to volume fraction of the fluid¿).



4.6 Displacement profile with respect to (magnetic parameters) M.



4.7. Temperature profile with respect heat source (α).

The plots above show the displacement at different parameters,

For the plot of Viscous drag, G_r Grashof number, volume fraction of fluid and magnetic parameters as the displacement increases the values of each parameters decreases.

For the viscous drag displacement with respect to viscous drag as displacement increases viscous drag reduces from 2.5 to 1.

For the Garshof number displacement against Grashof number, as displacement increases Grashof number decreases from 4 to 1.

For Volume fraction of fluid displacement with respect to volume fraction of the fluid, as displacement increases volume fraction of the fluid decreases from 0.8 to 0.2.

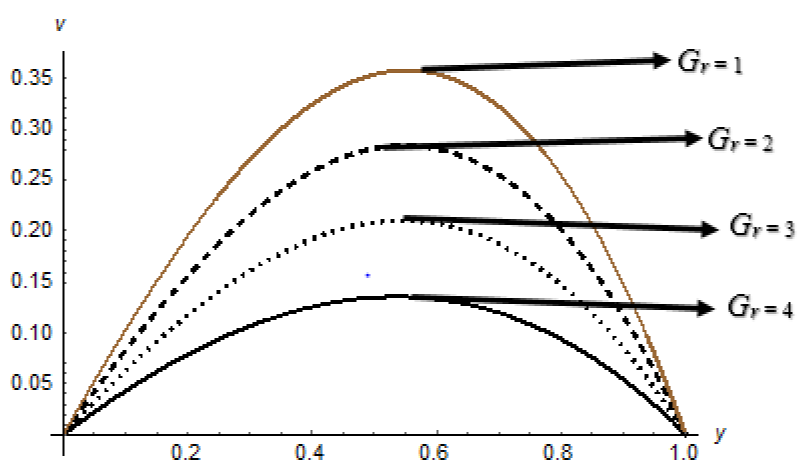
For the Magnetic parameters' displacement against Magnetic parameters, as displacement increases, magnetic parameters reduce from 4 to 1.

While for the Heat sources as displacement increases the heat sources also increases

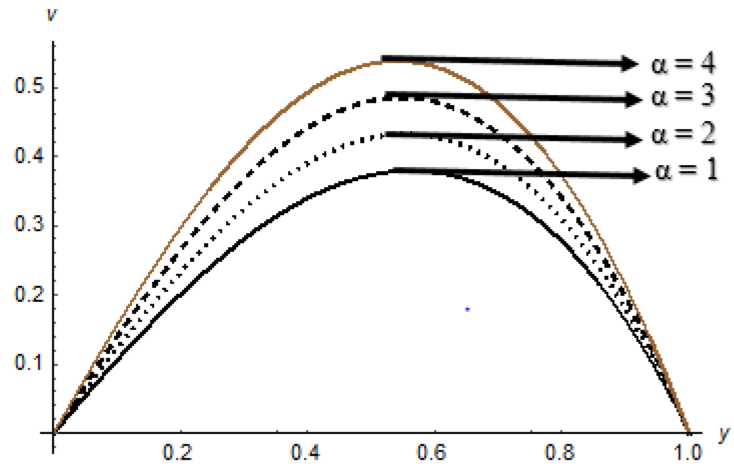
For heat source displacement against Heat source, as displacement increases Heat source increases from 1 to 4.

For heat source temperature against heat source, increase in temperature increase in heat source from 1 to 4.

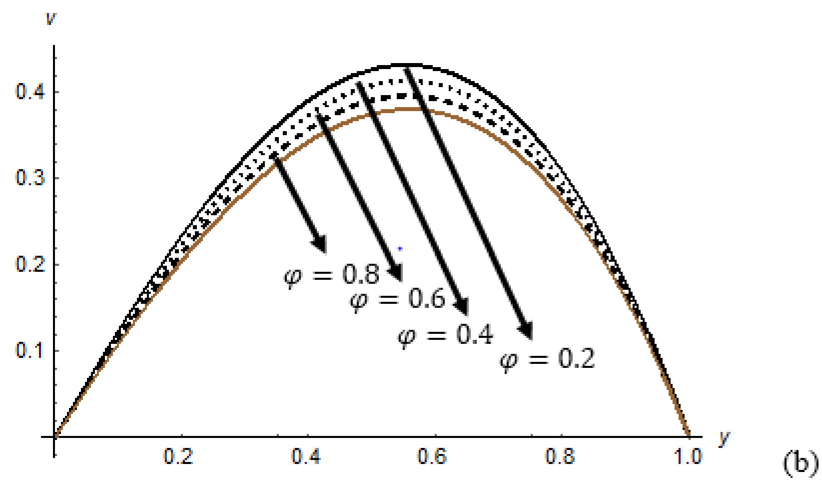
The following graphs representing the velocity profile of different parameters which includes heat source, G_r Grashof number, volume fraction of fluid, magnetic, temperature and bulk fluid.



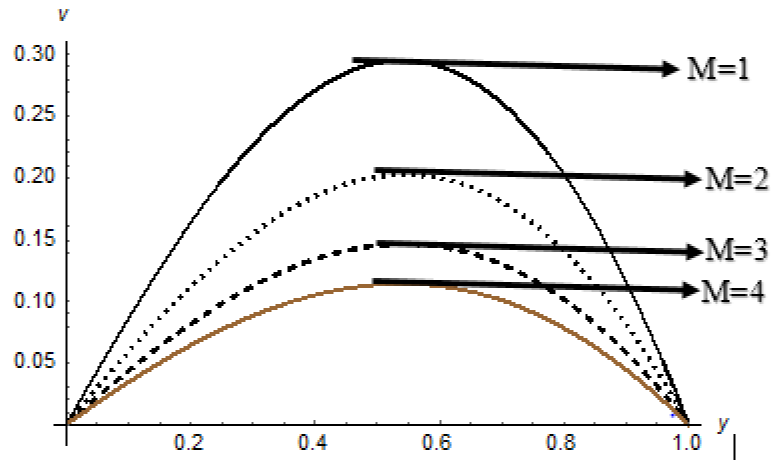
4.8 Velocity profile with respect to G_r(Grashof number).



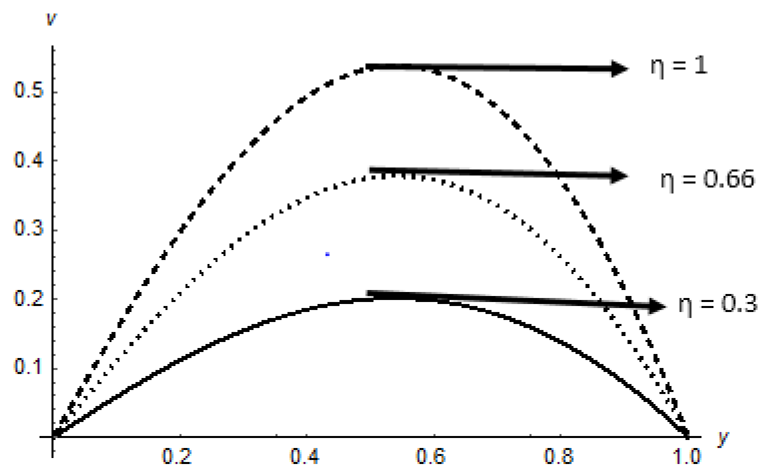
4.9. Velocity profile with respect to heat source(α).



4.10 velocity profile with respect to volume fraction of the fluid¿).



4.11 Velocity profile with respect to (magnetic parameters) M.



4.1 Velocity profile with respect to Bulk fluid viscosity ratio(η).

For the velocity graphs some parameters increased when there is an increase in Velocity this includes heat source and bulk fluid viscosity ratio while some parameters decreased when there is an increase in velocity which are Grashof number, volume fraction of the liquid and the magnetic parameters.

Velocity against Grashof number, as velocity increases Grashof number decreases from 4 to 1.

Velocity profile against heat source, velocity increase heat source increases from 1 to 4.

Velocity with respect to volume fraction of the liquid, as velocity increases the volume fraction of the liquid decreases from 0.8 to 0.2.

Velocity profile with respect to Magnetic parameters, as velocity increases magnetic parameters decreases from 4 to 1.

Velocity profile with respect to bulk fluid viscosity ratio, as velocity increases the bulk fluid viscosity ratio increases from 0.3 to 1.

CHAPTER FIVE

CONCLUSION

5.0 SUMMARY

This project analyzed the effect of entropy generation on the MHD flow through a vertical deformable porous layer. Furthermore, the expressions for the fluid velocity, solid displacement and temperature distribution were obtained analytically. Some useful observations were made and summarized as follows;

1. As the porosity of the two plates of the layer increases, the fluid velocity also increases as there is less solid to obstruct the flow consequently, causing the displacement of the solid to diminish since there is less drag on the solid properties.
2. The skin friction at ($y = 0$ and $y = 1$) shows different behavior with respect to volume fraction of the fluid φ and magnetic field M.

   In this study, we were able to analyzed and plot the graph of each parameters with the aid of mathematical package.

5.1 Recommendation

This research should aid the starting point for any theoretical, numerical and experimental studies to further understand MHD fluid flow. Further studies on the deformable porous layer should continue to enhance good understanding of MHD flow.

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