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Control and synchronization of chaos in RCL-shunted Josephson junction using backstepping design

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Abstract

This paper investigates the control and synchronization of chaotic dynamics in RCL-shunted Josephson junctions based on backstepping nonlinear control theory. The method consists of a recursive approach that interlaces the choice of a Lyapunov function with the control. The method was employed to eliminate the chaotic behavior exhibited by the RCL-shunted Josephson junctions as well as to achieve global asymptotic synchronization between a drive-response RCLSJ system with different system parameters. Numerical simulations have been employed to verify the effectiveness of the control scheme; while the closed loop systems with the control are perfectly modeled using SIMULINK block.

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Keywords: Josephson junctions; Chaos; Synchronization; Control; Backstepping

1. Introduction

The dynamical behavior of Josephson junctions (JJ) have for long attracted considerable research attention since Belykh, Pedersen and Soerensen published their work on chaos in Josephson junctions [1,2]. Thereafter, in 1980, Huberman et al. [3] presented numerical studies on chaos in JJ. Different models have been introduced to represent the JJ [4]. Amongst them are the Shunted linear resistive–capacitive junction (RCSJ) [4], the Shunted nonlinear resistive–capacitive junction (SNRCJ) [5], Shunted nonlinear resistive–capacitive–inductive junction (RCLSJ) [5–8] and the periodically modulated Josephson junction (PMJJ) [9,10]. The first two models of the Josephson junction contain two state variables and exhibit chaotic behavior with external sinusoidal signal, while the RCLSJ model which has been found to be very useful for high-frequency applications generate chaotic oscillations with external dc bias only. Wu and Li [10] recently carried out analytic and numerical investigations of the dynamics of periodically modulated Josephson junction (PMJJ) and showed that the PMJJ exhibits chaotic motion through the period-doubling cascade, when the amplitude of the modulation term is increased.

Beside the dynamics of a single Josephson junction, the dynamics of coupled Josephson junctions have also attracted research interest in the recent times [11–14]. For instance, intermittent synchronization has been reported in a resistively coupled chaotic JJ by Blackburn et al. [11]; while Dana et al. [13], recently investigated the synchronization behavior of uni-directionally coupled RCLSJ by means of a negative pulse forcing and observed intermittent synchronization. The robustness of the synchronization scheme to white noise was also established. A more recent study by Wang et al. [12], revealed a transition from synchronized state to quenching state in a mutually cou-

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pled JJ, with the two states co-existing for some coupling strength [12]. Indeed synchronization of chaotic systems can be interpreted in terms of the observer problem in control theory; and recent synchronization techniques address the problem of chaos synchronization based on control theory point of view; thus unifying the study of chaos control and chaos synchronization. In this direction, Ucar et al. [14] in a very recent paper extended the study of the dynamics of coupled Josephson junction by setting up the synchronization scheme for two-coupled RCLSJ in a master–slave configuration through suitably designed active controls.

The control and synchronization of chaotic systems have received increased research attention [15,16], since the classical work on chaos control was first presented by Ott et al. [17] in 1990, followed by the Pyragas time-delayed auto-synchronization control scheme [18]; and the pioneering work on the synchronization of identical chaotic systems evolving from different initial conditions was first introduced by Pecora and Carroll [19], the same year. The enormous research activities arising from the possible applications of chaos control and synchronization have motivated researchers to seek for various effective methods to achieve these goals. During the last one decade, the active control, which was originally proposed by Bai and Lonngren [20] has been extensively explored (For some recent applications, see for example Refs. [14,21–24]). In another development, backstepping design has been employed for controlling, tracking and synchronizing chaotic systems (see for example Refs. [25–29]); this is because backstepping design can guarantee global stability, tracking and transient performance for a broad class of strict-feedback nonlinear systems [30]. The technique is a systematic design approach and consists in a recursive procedure that skillfully interlaces the choice of a Lyapunov function and the control.

In a recent paper [31], we employed the backstepping approach to control intermittent chaotic transport in inertia ratchet that model the motion of a particle in an asymmetric periodic potential. In addition, we explored the property of backstepping scheme and proposed a simple active-backstepping approach for synchronizing different trajectories arising from different initial conditions in the ratchet model. In the present paper, we extend our investigation on backstepping control to RCLSJ model and present an active-backstepping scheme for the synchronization of two-coupled RCLSJ model, each subsystem evolving from different initial conditions and with different system parameters. The rest of the paper is organized as follows: In the next section, we describe briefly the RCLSJ model and present the backstepping designs in Section 3, together with numerical simulations. Section 4 concludes the paper.

2. The RCLSJ model

The RCLSJ model of JJ is described by the following set of first order differential equations:

$$\begin{aligned} \dot{x} &= y, \\ \dot{y} &= \frac{1}{\beta_C}[i - g(y)y - \sin(x) - z], \\ \dot{z} &= \frac{1}{\beta_L}(y - z), \end{aligned} \tag{1}$$

where the nonlinear damping function $g(y)$ is approximated by a current–voltage relation between the two junctions and is defined by

$$g(y) = \begin{cases} 0.366 & \text{if } |y| > 2.9, \\ 0.061 & \text{if } |y| \leqslant 2.9 \end{cases} \tag{2}$$

x, y, and z represent the phase difference, the voltage at the junction, and the inductive current, respectively. β_C and β_L are constants that represent capacitive and inductive values, respectively. i is the dc external current. This dissipative model has been shown to have an attractor in a bounded region. For instance, when the parameters are set as follows: $\beta_C = 2.6$ and $\beta_L = 0.707$ for the initial conditions: $(x(0), y(0), z(0)) = (0, 0, 0)$, the RCLSJ model exhibits chaotic dynamics for the dc external current in the region $1 < i < 1.3$ [5,7–9,13]. For the numerical results the system (1) and (2) is modeled using Matlab/SIMULINK block, Fig. 1. In Fig. 2a, we show a chaotic attractor for $i = 1.15$, while in Fig. 2b, we display a periodic attractor for $i = 1.5$. Our objective here is to design control law based on recursive backstepping approach that will eliminate the chaotic behavior and drive the system to a stable equilibrium point. Secondly, we would extend our previous investigation of synchronization behaviors of this system by means of a new active-backstepping approach, which we proposed recently [31].

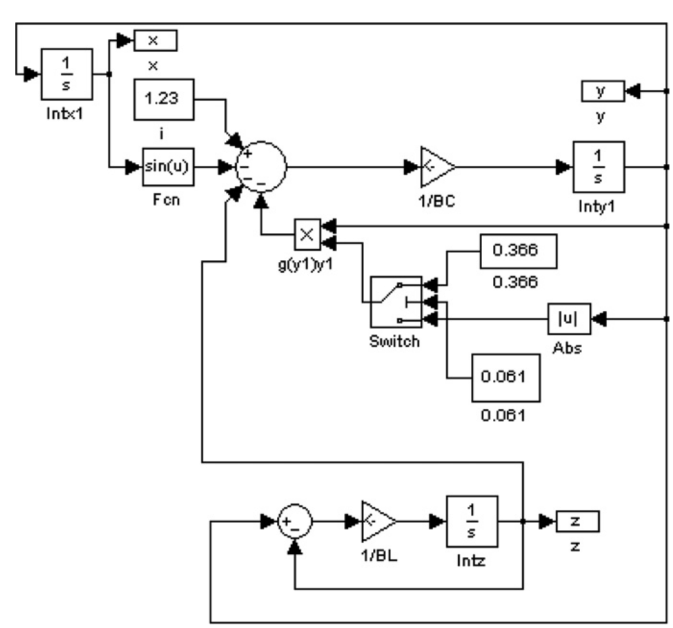

Fig. 1. SIMULINK block of the system (1) and (2).

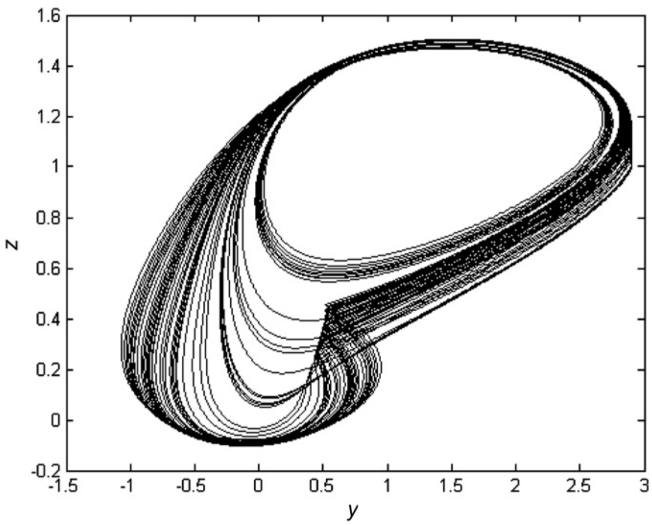

Fig. 2a. Phase portrait of y and z of the system (1) and (2) for $i = 1.15$.

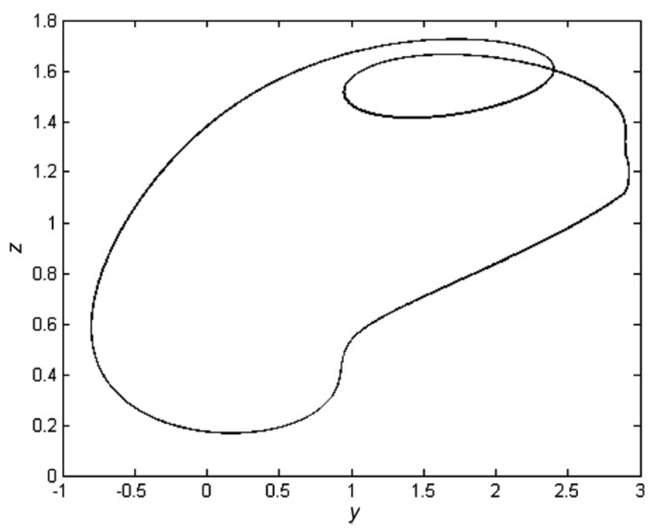

Fig. 2b. Phase portrait of y and z of the response system (9) for $i = 1.5$.

3. Backstepping design

3.1. Chaos control in RCLSJ model via backstepping design

Let us add a time dependent control input function $u(t)$ to system (1) to obtain

$$\begin{aligned} \dot{x} &= y, \\ \dot{y} &= \frac{1}{\beta_C}[i - g(y)y - \sin(x) - z] + u(t), \\ \dot{z} &= \frac{1}{\beta_L}(y - z). \end{aligned} \tag{3}$$

The goal is to design a $u(t)$ based on the recursive backstepping procedure that will drive the system (3) to a regular state. To achieve this goal, we define the differences between the current chaotic states x, y, z and anticipated regular states x_d, y_d, z_d, as the error functions, i.e.

$$\begin{aligned} e_x &= x - x_d, \\ e_y &= y - y_d, \\ e_z &= z - z_d. \end{aligned} \tag{4}$$

Let $x_d = 0$; while $y_d = c_1 e_x$ and $z_d = c_2 e_x + c_3 e_y$, are recursively introduced and c_i $(i = 1, 2, 3)$ are arbitrary control parameters to be determined. Using the above definitions, we obtain the following error dynamic systems as follows:

$$\begin{aligned} \dot{e}_x &= c_1 e_x + e_y, \\ \dot{e}_y &= \frac{1}{\beta c}[i - g(y)(c_1 e_x + e_y) - \sin(e_x) - c_2 e_x - c_3 e_y - e_z] \\ &\quad - c_1(c_1 e_x - e_y) + u(t), \\ \dot{e}_z &= \frac{1}{\beta_L}[c_1 e_x + e_y - c_2 e_x - c_3 e_y - e_z] - c_2(c_1 e_x + e_y), \\ &\quad - c_2 \left\{ \frac{1}{\beta c}[i - g(y)(c_1 e_x + e_y) - \sin(e_x) \right. \\ &\quad \left. - c_2 e_x - c_3 e_y - e_z] - c_1(c_1 e_x - e_y) \right\}. \end{aligned} \tag{5}$$

The control problem is to stabilize the error dynamics (5) at the equilibrium $(0, 0, 0)$. Let us consider the following Lyapunov function for the error dynamics (5):

$$V = \frac{1}{2}(k_1 e_x^2 + k_2 e_y^2 + k_3 e_z^2). \tag{6}$$

Since the controller must be as simple as possible, we let the c_i's vanish, so that the system is stabilized at the origin. Using Eq. (5) in Eq. (6) we obtain the Lyapunov first derivative

$$\begin{aligned} \dot{V} &= k_1 e_x e_y + \frac{k_2 e_y}{\beta_C}[(i - g(y)e_y - \sin(e_x) - e_z) + u(t)] \\ &\quad + \frac{k_3 e_x}{\beta_L}(e_y - e_z) \end{aligned} \tag{7}$$

If we choose

$$u(t) = -[e_y + \frac{1}{\beta_c}(i - g(y)e_y - sin(e_x) - e_z)] \tag{8}$$

and $k_1 = k_3 = 0$; and $k_2 = 1$, then $\dot{V} = -e_y^2$ is negative definite and according to LaSalle–Yoshizawa's theorem, the equilibrium $(0, 0, 0)$ is globally asymptotically stable and the control problem is solved.

The closed loop system with the control (8) is modeled in Matlab/SIMULINK and shown in Fig. 3.

We numerically simulated the closed loop system (1) for $0 \leqslant t \leqslant 600$ with the control signal defined in Eq. (8) activated for $t \geqslant 300$. The parameters of the system are set as before. However, the dc external current is fixed at $i = 1.15$ to ensure chaotic behavior as in Fig. 2a. Figs. 4 show the time response of the system variable when the control, u, also plotted in Fig. 4b is activated at $t = 300$. Clearly, the impact of the control is to drive the system to the desired equilibrium point $(0, 0, 0)$. In

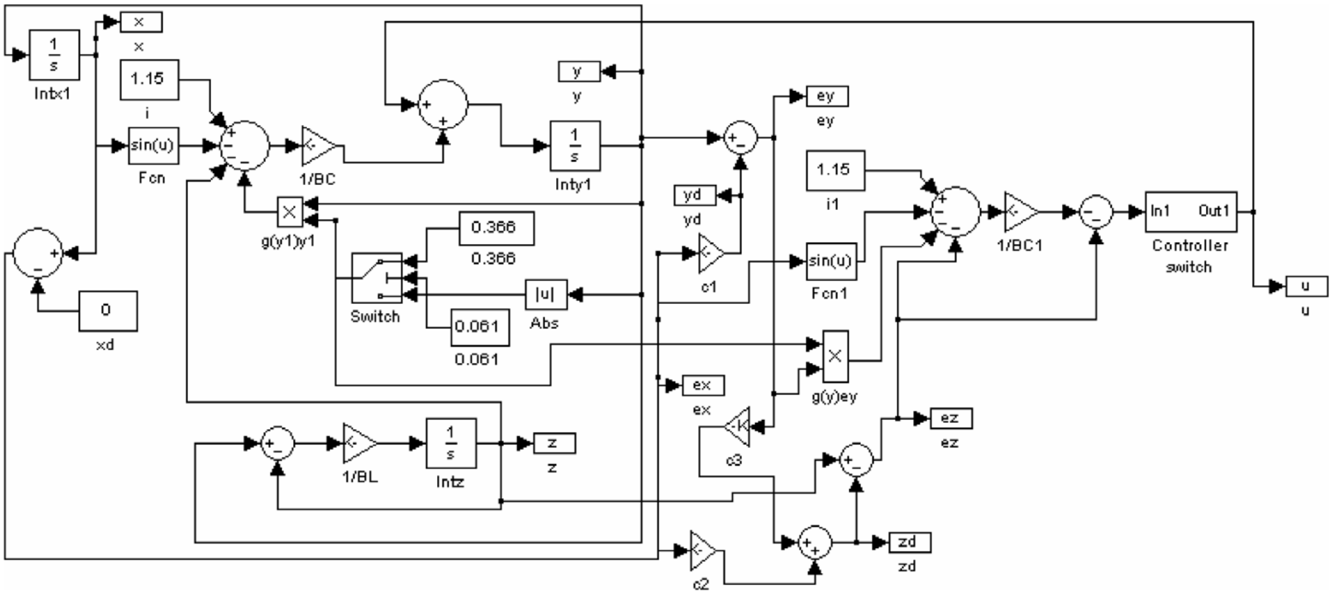

Fig. 3. SIMULINK block of the system (1) and (2) with the controller determined in Eq. (8).

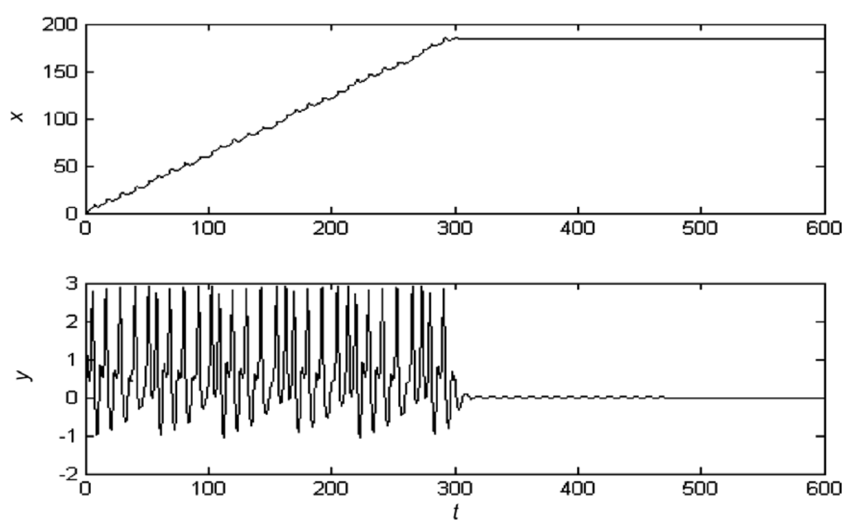

Fig. 4a. The time response of the system states x and y, where the control signal activated at $t = 300$ s.

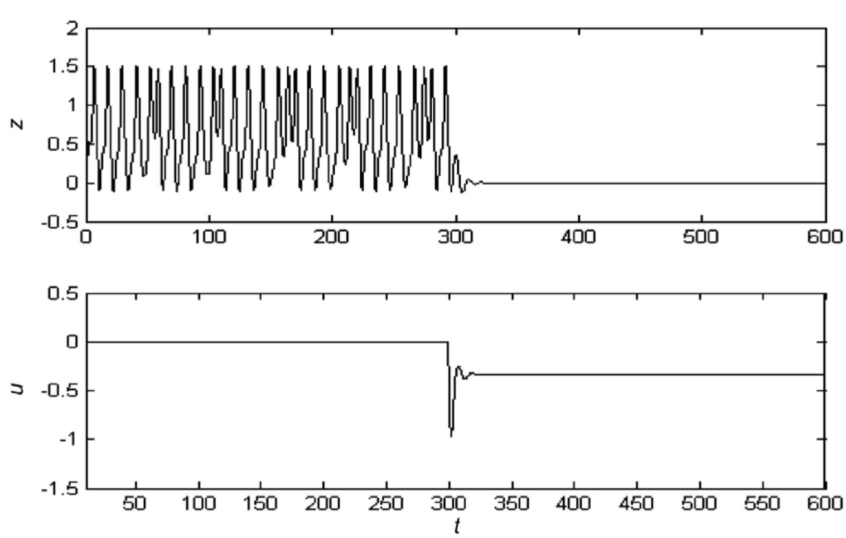

Fig. 4b. The time response of the system states z and control signal u, where the control signal activated at $t = 300$ s.

addition, the convergence of the error dynamics is illustrated in Fig. 5. Obviously, the simulation results presented in Figs. 4 and 5 confirm that the chaotic attractor has been controlled.

3.2. Synchronization of two-coupled RCLSJ models via active-backstepping

Very recently, we proposed an active-backstepping control based synchronization scheme for controlling two different trajectories arising from different initial conditions in an inertia ratchet [31]. Here, we extend the application of this technique to the RCLSJ model evolving from different initial conditions and with different system parameters as in [14]. Let us assume that system (1) with the state initial conditions $(x(0), y(0), z(0)) = (0, 0, 0)$ is the drive RCLSJ and considering another system evolving from different initial conditions $(x_2(0), y_2(0), z_2(0)) = (1, -1, 1)$ and with a different system parameter, $\tilde{i}$ which we assume to be the response system given by

$$\begin{aligned} \dot{x}_2 &= y_2 + u_1(t), \\ \dot{y}_2 &= \frac{1}{\beta_C}[\tilde{i} - g(y_2)y_2 - \sin(x_2) - z_2] + u_2(t), \\ \dot{z}_2 &= \frac{1}{\beta_L}(y_2 - z_2) + u_3(t), \end{aligned} \tag{9}$$

where $u_1(t)$, $u_2(t)$, and $u_3(t)$ are the control inputs.

Defining the error states between the drive-response systems as

$$\begin{aligned} e_x &= x_2 – x, \\ e_y &= y_2 – y, \\ e_z &= z_2 - z \end{aligned} \tag{10}$$

we obtain the following error dynamics system:

$$\begin{aligned} \dot{e}_x &= e_y + u_1(t), \\ \dot{e}_y &= \frac{1}{\beta c}(\tilde{i} - i) - \frac{1}{\beta c}[g(y_2)y_2 - g(y)y + sin(x_2) - sin(x)] \\ &\quad - \frac{1}{\beta c}e_z + u_2(t), \\ \dot{e}_z &= \frac{1}{\beta_L}(e_y - e_x) + u_3(t). \end{aligned} \tag{11}$$

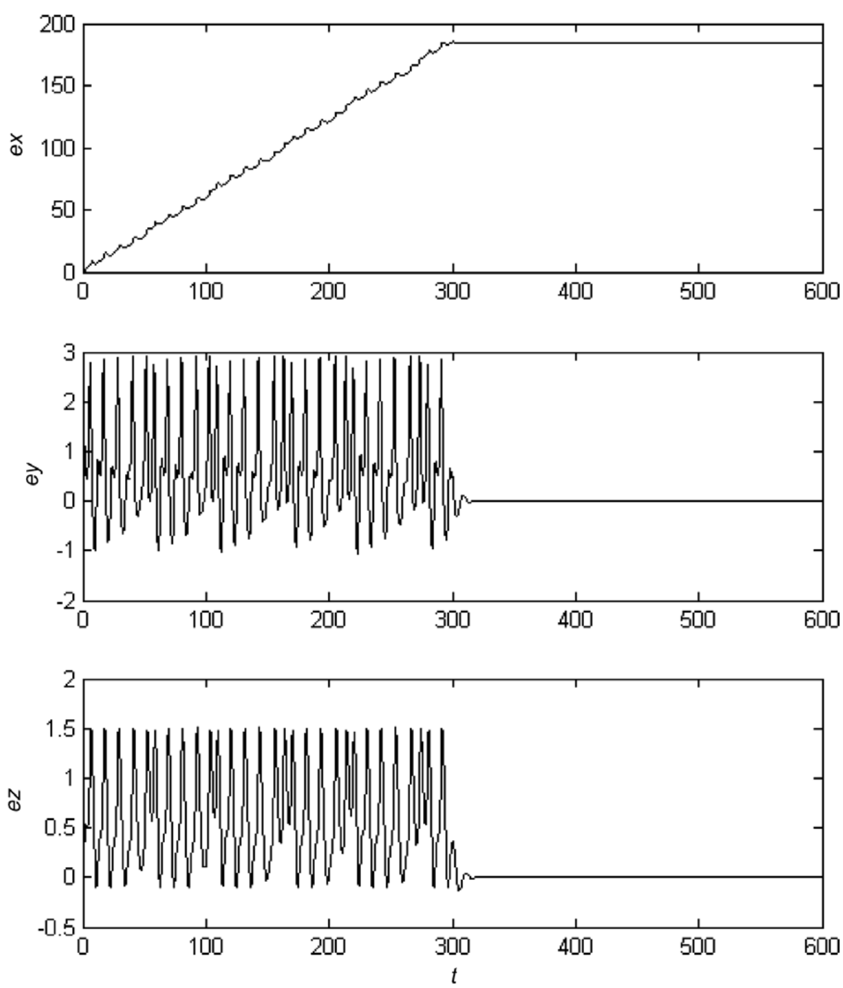

Fig. 5. The time response of the error dynamics defined in (4); e_x, e_y and e_z where the control signal u defined in (8) is activated at $t = 300$ s.

Substituting Eq. (11) into the time derivative of the Lyapunov function (6), we have

$$\dot{V} = k_1 e_x[e_y + u_1(t)] + k_2 e_y\left\{\frac{1}{\beta_C}(\tilde{i} - i) - \frac{1}{\beta_C}[g(y_2)y_2 - g(y)y + \sin(x_2) - \sin(x)] - \frac{1}{\beta_C}e_z + u_2(t)\right\} + k_3 e_3\left[\frac{1}{\beta_L}(e_y - e_z) + u_3(t)\right]. \quad (12)$$

If we choose the control functions as follows:

$$u_1(t) = -(e_x + e_y),$$
$$u_2(t) = -e_y - \frac{1}{\beta_C}\{(\tilde{i} - i) - [g(y_2)y_2 - g(y)y + \sin(x_2) - \sin(x)] - e_z)\},$$
$$u_3(t) = -e_z - \frac{1}{\beta_L}(e_y - e_z) \quad (13)$$

and the k_i's $(i = 1, 2, 3) = 1$, then

$$\dot{V} = -e_x^2 - e_y^2 - e_z^2 \quad (14)$$

is negative definite and according to LaSalle–Yoshizawa theorem, the error dynamics (10) will converge to zero and remains globally asymptotically stable. Thus, the synchronization problem between the drive-response RCLSJ is solved.

In Fig. 6, we display the SIMULINK block implementation of the closed loop systems (1) and (9) with the control signals defined in (13).

In the numerical results that follow, we set $\beta_C = 2.6$ and $\beta_L = 0.707$; while the dc external currents for the drive system is $i = 1.15$ and for the response system is $\tilde{i} = 1.5$. Note that with these system parameters the drive RCLSJ exhibits the chaotic trajectory depicted in Fig. 2a and the response system exhibits the periodic trajectory shown in Fig. 2b. Fig. 7 shows the time evolution of the drive (x, y, z) and response (x_2, y_2, z_2) system states where the control signal defined in (13) has been activated at $t = 150$ s. The corresponding error dynamics defined in Eq. (10) is also

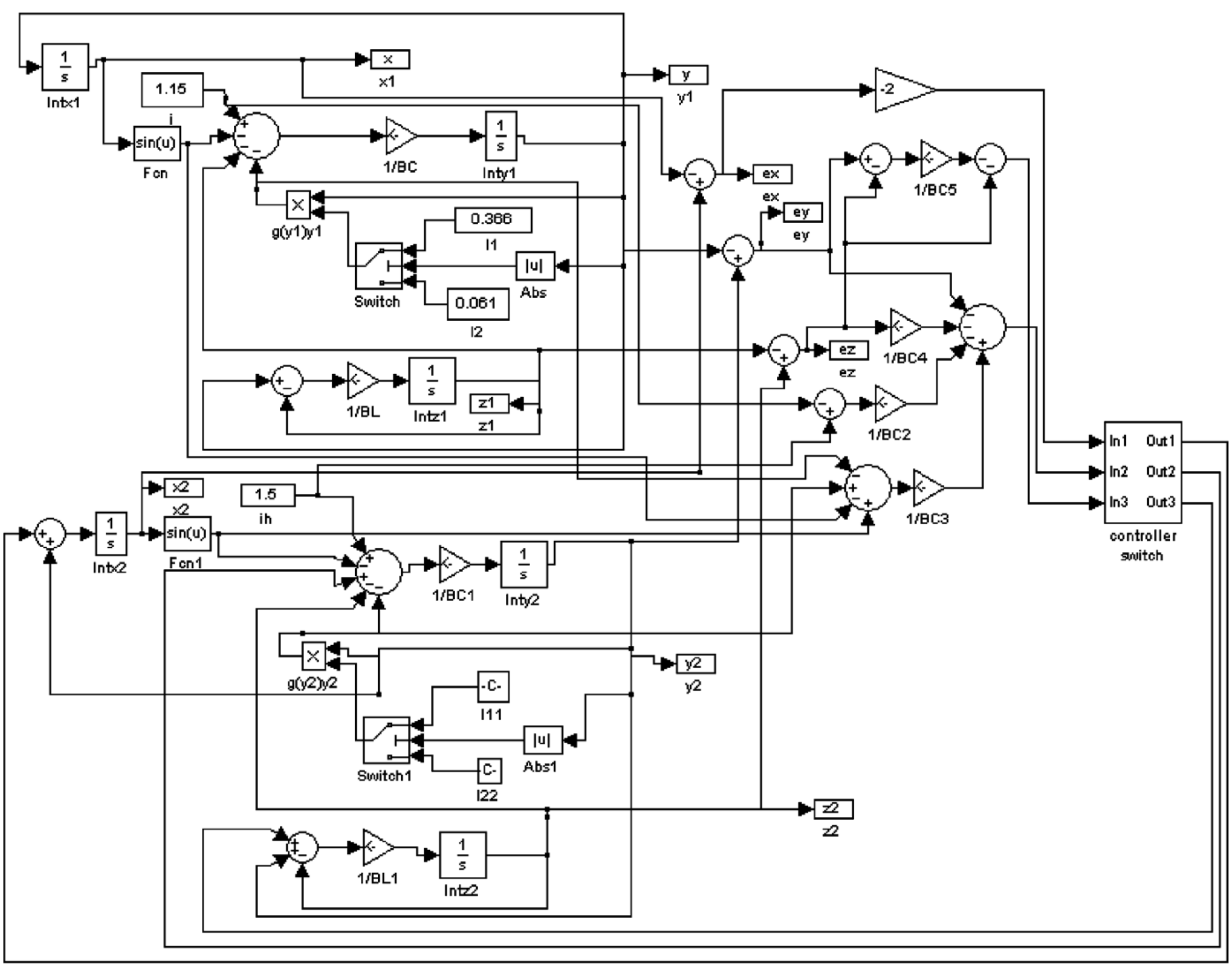

Fig. 6. SIMULINK block of the closed loop systems (1) and (9) with the control signal defined in (13).

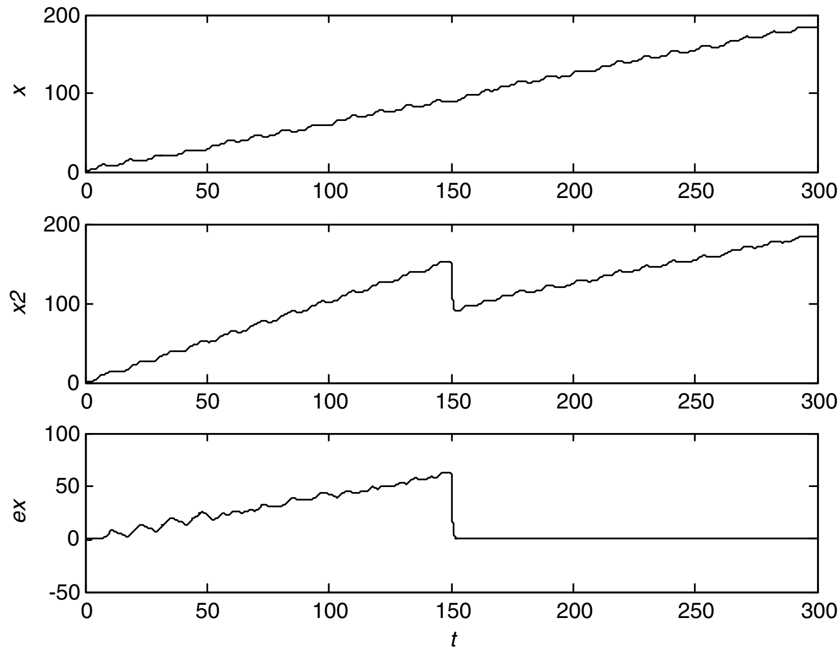

Fig. 7a. The time response of the drive system state x and response system state x_2 and the error $e_x = x_2 - x$ where the control signals define in (13) activated at $t = 150$ s.

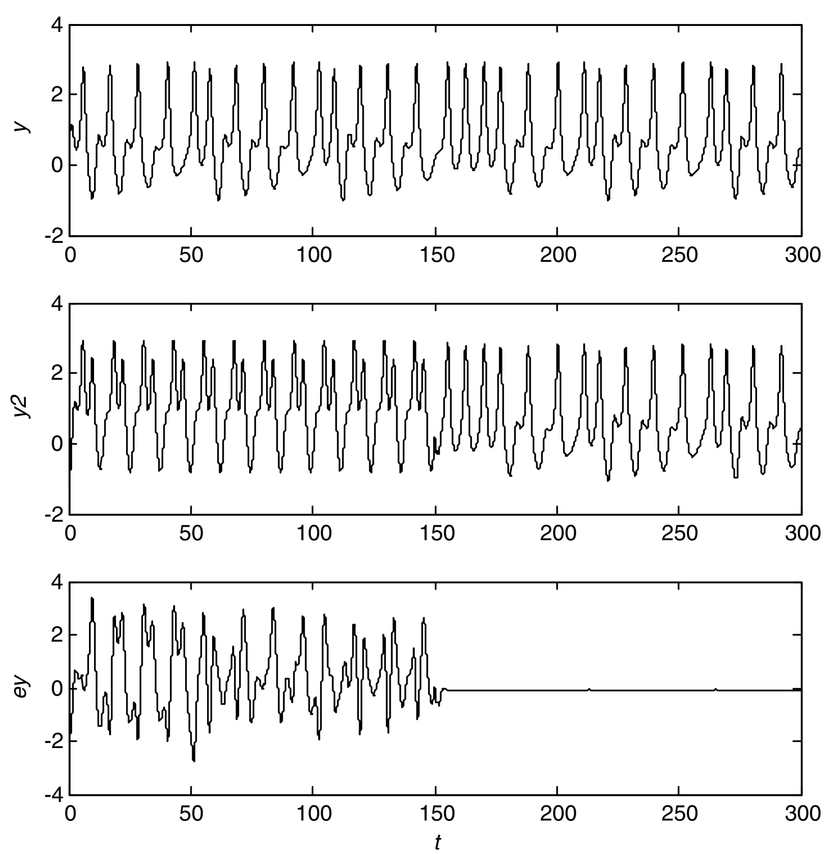

Fig. 7b. The time response of the drive system state y and response system state y_2 and the error $e_y = y_2 - y$ where the control signals define in (13) activated at $t = 150$ s.

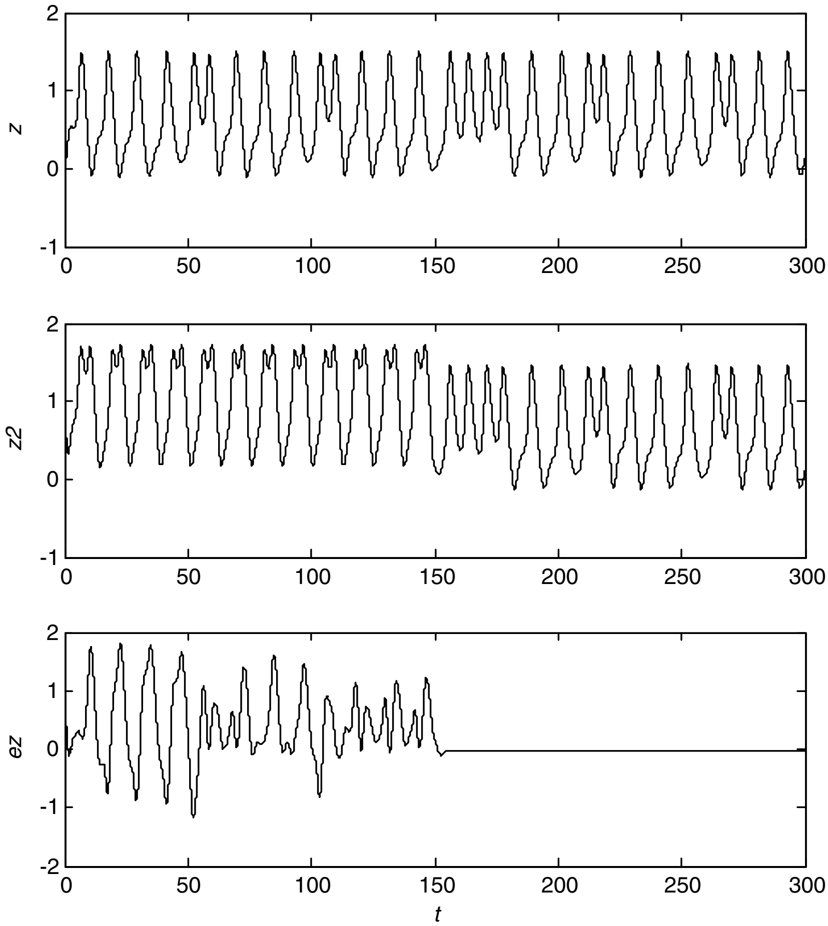

Fig. 7c. The time response of the drive system state z and response system state z_2 and the error $e_z = z_2 - z$ where the control signals define in (13) activated at $t = 150$ s.

displayed. It is very clear that the error dynamics $(e_x, e_y, e_z) \to 0$ as $t \to \infty$ as soon as the control is activated.

4. Concluding remarks

In this paper, we have investigated the control and synchronization of chaos in the RCLSJ model of the Josephson junction. The control of chaotic behavior in the Josephson junction is of significant importance because most researchers in Josephson junction often tend to avoid the region of chaotic behavior during application as a high-frequency oscillator [5–7]. Here, we have presented a technique of driving the JJ to stable regular state. Thus, in practice, the chaotic region need not be avoided, but could be exploited, for instance in secure communication through the mechanism of drive-response synchronization which we have also shown using the backstepping design. The backstepping design that we employed provides an efficient recursive approach, which is interlaced with the choice of appropriate control and guarantee global stability and transient performance as have been illustrated with several numerical simulations. Besides, the Matlab/SIMULINK models of the RCLSJ system have been demonstrated, suggesting that the approach can be well implemented.

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