



Entropy generation analysis for a reactive couple stress fluid flow through a channel saturated with porous material



Samuel O. Adesanya^{a,*}, Semiu O. Kareem^b, John A. Falade^b, Samson A. Arekete^c

^a Department of Mathematical Sciences, Redeemer's University, Ede, Nigeria

^b Department of Physical Sciences, Redeemer's University, Ede, Nigeria

^c Department of Computer Science, Redeemer's University, Ede, Nigeria

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ABSTRACT

In this work, the second law analysis for a reactive couple stress fluid flowing steadily through a channel filled with porous material is investigated. Based on perturbation method, analytical expressions for fluid velocity and temperature are derived and used to compute entropy generation rate, irreversibility distribution ratio in the flow field and results are presented graphically and discussed.

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1. Introduction

There has been a renewed interest in the study of reactive fluid flow problems due to its importance in many reservoir and automobile engineering applications. Some of these studies include: Makinde [1] examined the steady-state Brinkman model for a strongly exothermic reaction of a viscous combustible material in a channel filled with a saturated porous medium under Arrhenius kinetics. Makinde [2] analyzed the thermal stability of a reactive third-grade liquid flowing steadily between two parallel plates with symmetrical convective cooling at the walls in which the system exchange heat with the ambient following Newton's cooling law. Makinde [3] studied the thermal stability of a reactive viscous combustible fluid flowing steadily through a channel filled with a saturated porous medium. It was assumed that the exothermic system exchanged heat with the ambient following Newton's cooling law.

In reality, many real fluid contains nanoparticles in form of polymer additives in case of lubricants, red blood cells in case of blood while some contains metals like (Al, Cu), oxides (Al_2O_3) in enhancing heat transfer processes. Based on these applications, quite a good number of researchers have studied various aspects of

the nanofluid problems. Noticeably among these researchers is Stokes [4] who proposed a Couple stress fluid theory as a simple generalization of the classical Newtonian model. The modified model took into account the presence of couple stresses, body couples and non-symmetric stress tensor. Based on these, Muthuraj et al. [5] developed a model to examine the effect of chemical reaction on magnetohydrodynamic (MHD) mixed convective heat and mass transfer flow of a couple-stress fluid in vertical porous space in the presence of a temperature dependent heat source with traveling thermal waves. Kaladhar and Srinivasacharya [6] examined the mixed convective flow of couple stress fluid through a circular annulus undergoing first order chemical reaction taking Soret and Dufour effects into consideration. Srinivasacharya and Kaladhar [7] investigated the effects of Hall and ion-slip on electrically conducting couple stress fluid flow between two circular cylinders in the presence of a temperature dependent heat source. Hayat et al. [8] analyzed the unsteady three-dimensional flow of couple stress fluid over a stretched surface. Analysis included the mass transfer and chemical reaction. Other related work on couple stress fluid can be found in Refs. [9–10].

Since heat transfer to non-Newtonian couple stress fluid is irreversible hence entropy is generated continuously due to exchange of kinetic energy and fluid friction within the flow channel. This leads to the destruction of the available energy for work in the thermo-fluid system. To monitor the effectiveness and performance of a fluid system exchanging heat between heat reservoirs,

* Corresponding author.

E-mail address: adesanyaolumide@yahoo.com (S.O. Adesanya).

the second law of thermodynamics will be used as proposed by Bejan [11,12]. Similar analysis have been conducted in Refs. [13–27] with the intention to upgrade the efficiency of some thermo-fluid system. In the class of couple stress fluid, not much has been achieved. For instance, Egunjobi and Makinde [28] investigated the entropy generation in the flow of an inert couple stress fluid flowing steadily through a non-Darcian porous medium. Also, Adesanya and Makinde [29–31] reported the irreversibility analysis in the flow of couple stress fluid under different flow geometries.

Motivated by studies in Refs. [4–31], the objective of the present study is to analyze the thermal performance of a wide range of chemically reactive couple stress fluid flowing steadily through a channel filled with porous material. As against the linear dependence of internal heat generation on temperature, the generalized

In Equations (1)–(4), T is the absolute temperature, P is the fluid pressure, T_0 is the geometry wall temperature, k is the thermal conductivity of the fluid, K is the porous permeability of the medium, Q is the heat of reaction, A is the rate constant, E is the activation energy, R is the universal gas constant, C_0 is the initial concentration of the reactant species, h is the channel width, (x, y) the distance measured in the axial and normal directions, respectively, and μ is the combustible material dynamic viscosity coefficient. η is the coefficient of couple stresses and U is the fluid characteristic velocity.

Observe that in (4), the first term represents the heat transfer rate while the last three terms accounts for viscous dissipations and effect of porosity. Introducing the following dimensionless parameters and variables,

$$y = \frac{y'}{h}, u = \frac{u'}{UM}, \theta = \frac{E(T - T_0)}{RT_0^2}, \lambda = \frac{QEAC_0 h^2 e^{-E/RT_0}}{RT_0^2 k}, M = -\frac{h^2}{\mu U} \frac{dP}{dx},$$

$$\epsilon = \frac{RT_0}{E}, \delta = \frac{U^2 \mu M^2 e^{E/RT_0}}{QAC_0 h^2}, \gamma = \frac{h}{l}, l = \sqrt{\frac{\eta}{\mu}}, \beta = \sqrt{\frac{1}{Da}}, Da = \frac{K}{h^2}, N_s = \frac{h^2 E^2 E_G}{kR^2 T_0^2}$$

nonlinear Frank-Kameneskii internal heat generation model will be used since heat liberation during chemical reactions depends largely on the initial concentration of the reacting species in the combustion zones. The paper is organized as follows; in the following section the problem is formulated and non-dimensionalized. The analytical solution by perturbation method is developed in Section 3. The graphical results are presented and the effects of the parameters are discussed in Section 4 followed by concluding remarks in Section 5.

2. Mathematical analysis

Consider the hydrodynamically and thermally developed unidirectional flow of a combustible couple stress fluid through an impermeable channel with isothermal boundaries placed at $y = 0$ and $y = a$, filled with a homogeneous and isotropic porous medium. The flow is induced by a constant pressure gradient in the direction of flow. The governing momentum and energy equations for the couple stress fluid flow can be written as [1].

$$0 = -\frac{dP}{dx} + \mu \frac{d^2 u'}{dy^2} - \frac{\mu u'}{K} - \eta \frac{d^4 u'}{dy^4}, \quad (1)$$

$$0 = \frac{d^2 T}{dy^2} + \frac{QC_0 A}{k} e^{-\frac{E}{RT}} + \frac{\mu}{k} \left(\frac{du}{dy} \right)^2 + \frac{\mu u^2}{kK} + \frac{\eta}{k} \left(\frac{d^2 u}{dy^2} \right)^2 \quad (2)$$

in (1)–(2), the last terms represent the effect of couple stresses [29–31]. The appropriate conditions at the walls are

$$\theta(0) = \theta(h) = 0, \\ u(0) = u''(0) = 0 = u''(h) = u(h). \quad (3)$$

The appropriate expression for the entropy generation in the couple stress fluid flow can be written as

$$E_G = \frac{k}{T_0^2} \left(\frac{dT}{dy} \right)^2 + \frac{\mu}{T_0} \left(\frac{du}{dy} \right)^2 + \frac{\mu u^2}{T_0 K} + \frac{\eta}{T_0} \left(\frac{d^2 u}{dy^2} \right)^2 \quad (4)$$

we obtain the following boundary-valued-problems

$$1 + \frac{d^2 u}{dy^2} - \beta^2 u - \frac{1}{\gamma} \frac{d^4 u}{dy^4} = 0; u(0) = u''(0) = u''(1) = u(1) = 0 \quad (6)$$

$$\frac{d^2 \theta}{dy^2} + \lambda \left\{ e^{\frac{\theta}{1+\epsilon\theta}} + \delta \left[\left(\frac{du}{dy} \right)^2 + \frac{1}{\gamma} \left(\frac{d^2 u}{dy^2} \right)^2 + \beta^2 u^2 \right] \right\} = 0, \theta(0) = \theta(1) = 0, \quad (7)$$

and the dimensionless entropy generation is given by

$$N_s = \left(\frac{d\theta}{dy} \right)^2 + \frac{\delta \lambda}{\epsilon} \left(\left(\frac{du}{dy} \right)^2 + \frac{1}{\gamma} \left(\frac{d^2 u}{dy^2} \right)^2 + \beta^2 u^2 \right) \quad (8)$$

Observe that the solutions of Equations (6)–(8) exist if and only if $\gamma \neq 0$. In Equations (1)–(8), λ , ϵ , δ , β , Da represent the Frank-Kameneskii parameter, activation energy parameter, the viscous heating parameter, the porous medium permeability parameter, the Darcy number, respectively, γ is the couple stress inverse parameter, l is a function of molecular dimension of the fluid, E_G , N_s , Be represent the dimensional, dimensionless entropy generation and Bejan number respectively.

3. Method of solution

To obtain an analytical solution of the dimensionless Equations (6)–(7), it is convenient to assume a power series expansion in the nonlinear Frank-Kameneskii parameter for the temperature field and a similar series expansion in the couple stress parameter for the solution of the velocity field in the form

$$u(y) = \sum_{n=0}^{\infty} u_n(y) \gamma^n, \quad \theta(y) = \sum_{n=0}^{\infty} \theta_n(y) \lambda^n. \quad (9)$$

Substituting (9) in the boundary value problems (6)–(7) and equating coefficients, we obtain the following equations

$$\begin{aligned}
 O(\gamma)^0 &: -\frac{d^4 u_0}{dy^4} = 0; \quad u_0(0) = u_0''(0) = u_0''(1) = u_0(1) = 0 \\
 O(\gamma)^1 &: 1 + \frac{d^2 u_0}{dy^2} - \beta^2 u_0 - \frac{d^4 u_1}{dy^4} = 0; \quad u_1(0) = u_1''(0) = u_1''(1) = u_1(1) = 0 \\
 O(\gamma)^2 &: \frac{d^2 u_1}{dy^2} - \beta^2 u_1 - \frac{d^4 u_2}{dy^4} = 0; \quad u_2(0) = u_2''(0) = u_2''(1) = u_2(1) = 0 \\
 &\dots \\
 O(\gamma)^n &: \frac{d^2 u_{n-1}}{dy^2} - \beta^2 u_{n-1} - \frac{d^4 u_n}{dy^4} = 0; \quad u_n(0) = u_n''(0) = u_n''(1) = u_n(1) = 0
 \end{aligned}
 \tag{10}$$

and

$$\begin{aligned}
 O(\lambda)^0 &: \frac{d^2 \theta_0}{dy^2} = 0; \quad \theta_0(0) = \theta_0(1) = 0 \\
 O(\lambda)^1 &: \frac{d^2 \theta_1}{dy^2} + e^{\frac{\theta_0}{1+\epsilon\theta_0}} + \frac{\delta}{\gamma} \left(\frac{d^2 u}{dy^2}\right)^2 + \delta\beta^2 u^2 + \delta\left(\frac{du}{dy}\right)^2 = 0; \quad \theta_1(0) = \theta_1(1) = 0 \\
 O(\lambda)^2 &: \frac{d^2 \theta_1}{dy^2} + \frac{\theta_1}{(1+\epsilon\theta_0)^2} e^{\frac{\theta_0}{1+\epsilon\theta_0}} = 0; \quad \theta_2(0) = \theta_2(1) = 0 \\
 &\dots
 \end{aligned}
 \tag{11}$$

using the DSolve algorithm in a computer algebra software package (MATHEMATICA), we obtained the first few terms of the above Equations (10) and (11) in the form

$$u(y) = \sum_{n=0}^m u_n(y)\gamma^n, \quad \theta(y) = \sum_{n=0}^r \theta_n(y)\lambda^n \tag{12}$$

as the analytical solution in which m and r are the truncation point. Due to the large size of the computational solution only the graphical solutions of (12) are presented as Figs. 1–5.

Next, we focus our attention on the entropy generation rate within the flow channel. To do this, the series solutions obtained in (12) are substituted in the expression for the entropy generation rate (8) and the results are presented in Figs. 6–9.

Moreover, if we set

$$N_1 = \left(\frac{d\theta}{dy}\right)^2, \quad N_2 = \frac{\delta\lambda}{\epsilon} \left(\left(\frac{du}{dy}\right)^2 + \frac{1}{\gamma} \left(\frac{d^2 u}{dy^2}\right)^2 + \beta^2 u^2 \right) \tag{13}$$

as the irreversibility due to heat transfer and fluid friction respectively, then the Bejan number that measures the ratio of heat transfer and fluid friction within the flow channel can be written as

$$Be = \frac{N_1}{N_1 + N_2} = \frac{1}{1 + \Phi}, \quad \Phi = \frac{N_2}{N_1} \tag{14}$$

from (14), it is evident that

$$Be = \begin{cases} 0, & N_2 \gg N_1 \\ 0.5 & N_1 = N_2 \\ 1 & N_1 \gg N_2 \end{cases} \tag{15}$$

please refer to Figs. 10–12 for the graphical result of the irreversibility ratio given in (14).

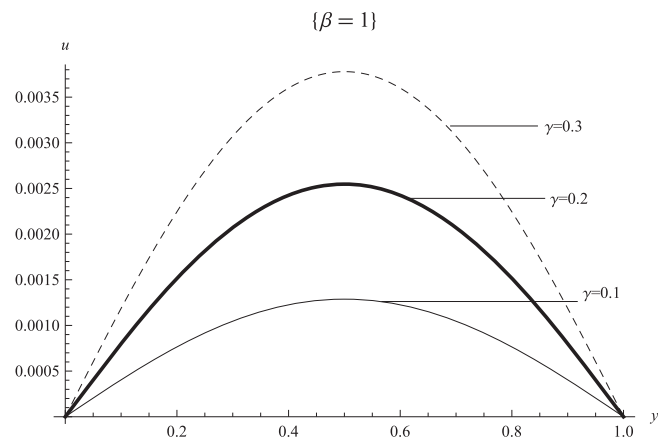


Fig. 1. Effect of γ on $u(y)$.

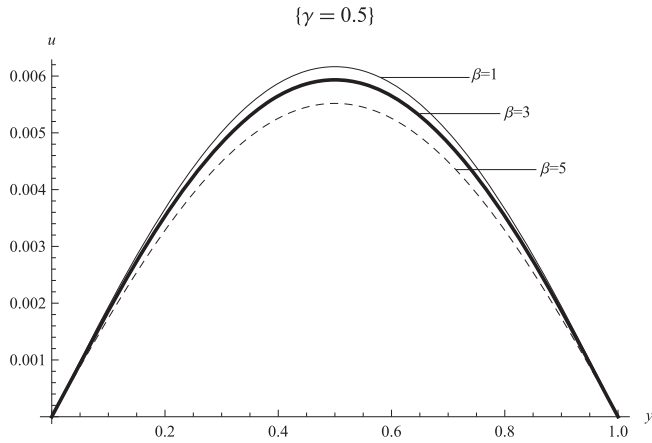


Fig. 2. Effect of β on $u(y)$.

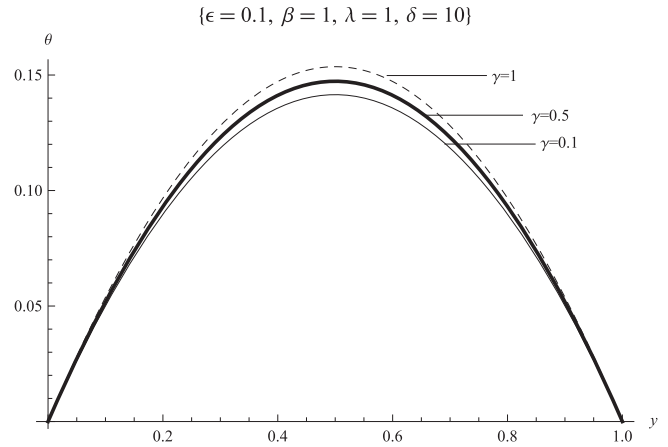


Fig. 5. Effect of γ on $\theta(y)$.

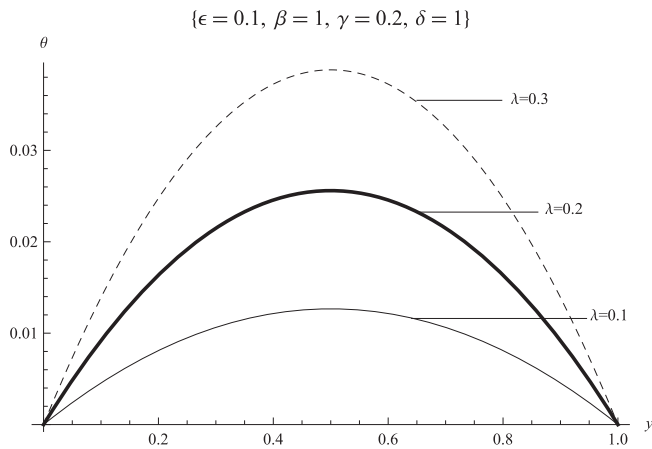


Fig. 3. Effect of λ on $\theta(y)$.

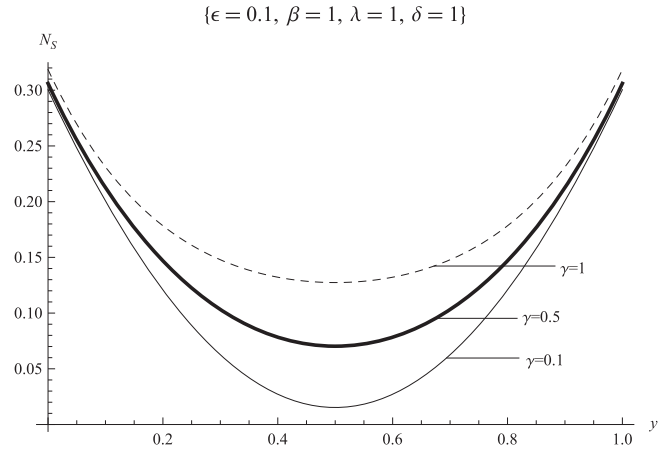


Fig. 6. Effect of γ on N_s .

4. Results and discussion

The graphical solution of the problem is hereby presented in this section to show the effect of pertinent parameters on the fluid flow. To show the reliability of the series solution (9), the series solution for the nonlinear boundary value problem (7) is shown to be

convergent in Table 1 for the given parameter values while the exact solution of the linear dimensionless boundary-value-problem (6) is compared with the analytical solution obtained through perturbation method in Table 2.

Fig. 1 depicts the effect of the couple stress inverse parameter on the reactive fluid flow through the porous medium. From the plot, it

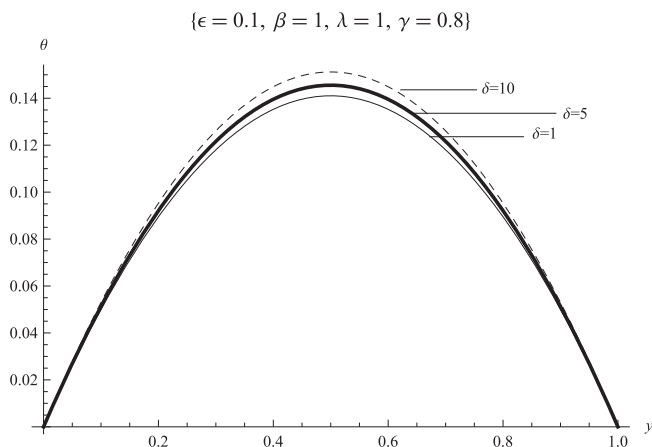


Fig. 4. Effect of δ on $\theta(y)$.

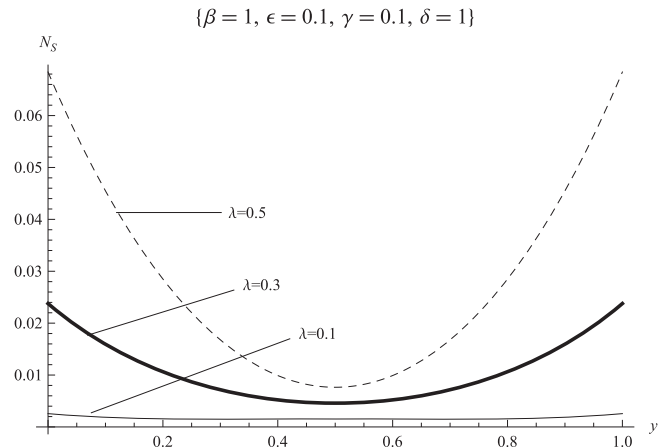


Fig. 7. Effect of λ on N_s .

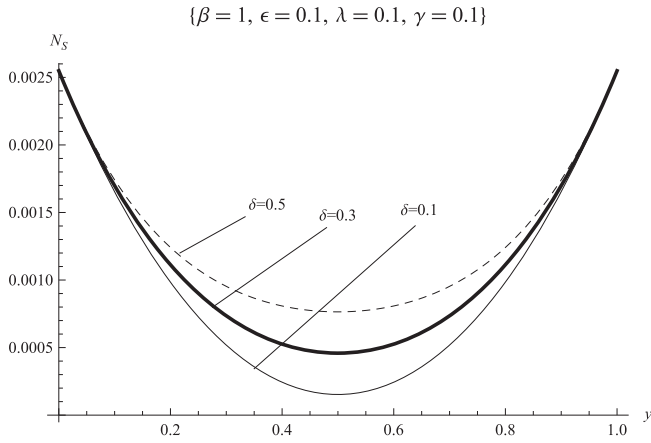


Fig. 8. Effect of δ on N_s .

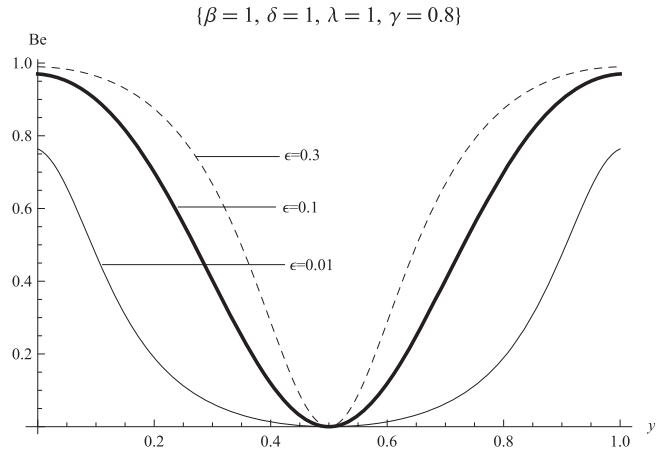


Fig. 11. Effect of ϵ on Be .

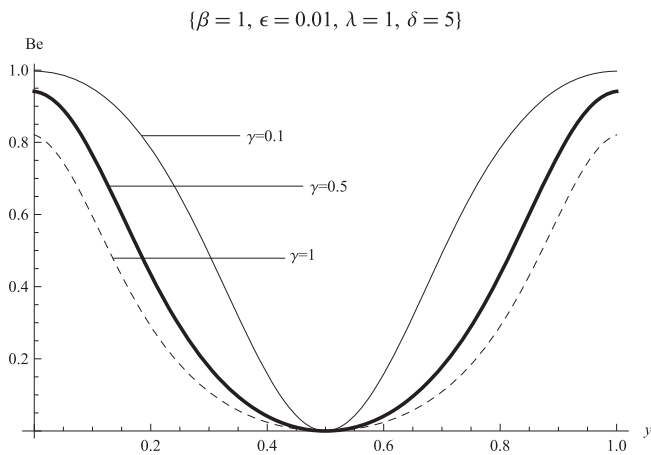


Fig. 9. Effect of δ on Be .

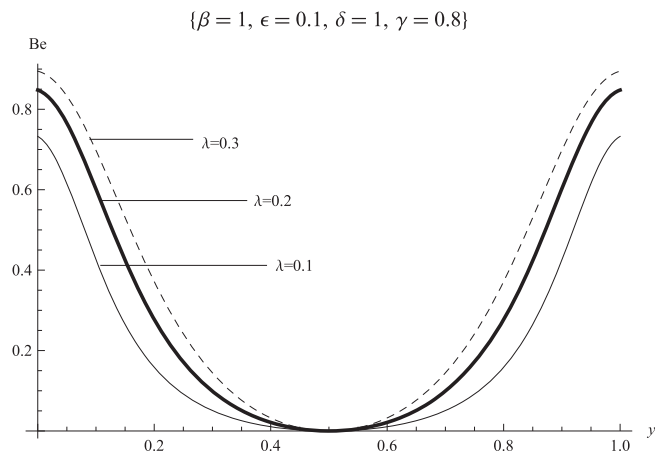


Fig. 12. Effect of λ on Be .

is observed that an increase in the couple stress inverse parameter enhances the fluid flow. This is physically true due to decrease in the dynamic viscosity of the fluid. However, the decrease in the couple stress inverse parameter is associated with the increase in dynamic viscosity of the fluid in which the fluid thickens. Fig. 2 represents the effect of variations in the porous permeability

parameter on the fluid flow velocity as observed from the plot, an increase in the porous permeability parameter leads to decrease in the fluid flow velocity. This is so since porous permeability of the medium decreases with increase in the porous permeability parameter and the channel becomes homogenous only if porous permeability becomes infinite.

Fig. 3 shows the influence of Frank-Kamenetskii parameter on the fluid temperature distribution within the flow channel. As observed from the graph, an increase in the Frank-Kamenetskii (nonlinear internal heat generation) parameter leads to an increase in the fluid temperature distribution within the flow channel. This is true since the initial concentration of the reagent increases with the Frank-Kamenetskii parameter provided the heat generation do not exceed the heat dissipation which could lead to spontaneous heating of the fluid. Fig. 4 depicts the effect of viscous

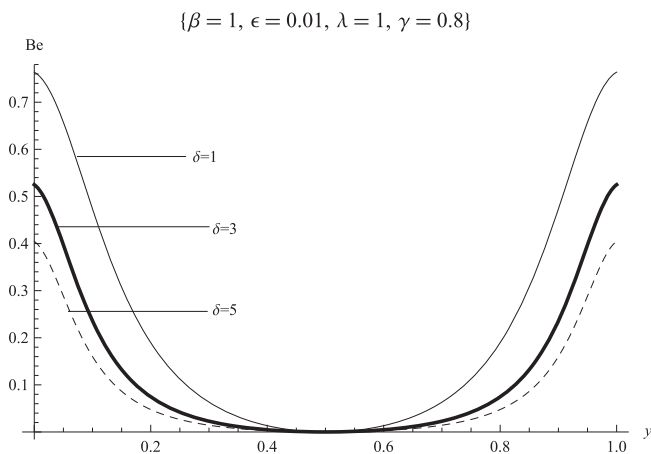


Fig. 10. Effect of δ on Be .

Table 1
Rapid convergence of the series solutions for $\epsilon = 0.1, \beta = 1, \gamma = 0.1 = \delta = \lambda, y = 0.1$.

n	$\theta_n(y)$	$\sum_{n=0}^m \theta_n(y) \lambda^n$	$u_n(y)$	$\sum_{n=0}^m u_n(y) \gamma^n$
0	0	0	0	0
1	0.045004100	0.00450041	0.0040875	0.00040875
2	0.004087930	0.00454129	-0.000451234	0.000404238
3	0.000575772	0.00454186	0.0000503003	0.000404288
4	0.0000960873	0.00454187	-5.6123E-06	0.000404287
5	0.0000175799	0.00454187	6.26254E-07	0.000404287

Table 2
Convergence of $u(y)$ when $m = 10$, $\beta = 0.1$, $\gamma = 1$.

$u(y)$	Exact solution	Perturbation method	Absolute error
0	0	0	0
0.1	0.00371505	0.00371505	4.28012E-13
0.2	0.00702528	0.00702528	8.05942E-13
0.3	0.00961357	0.00961357	1.09644E-12
0.4	0.01125560	0.01125560	1.29995E-12
0.5	0.01181780	0.01181780	1.35280E-12
0.6	0.01125560	0.01125560	1.29363E-12
0.7	0.00961357	0.00961357	1.09612E-12
0.8	0.00702528	0.00702528	8.08630E-13
0.9	0.00371505	0.00371505	4.21368E-13
1	0	-5.21948E-20	5.21948E-20

heating of the fluid on the fluid temperature distribution. From the plot, an increase in viscous heating parameter enhances the fluid temperature due to additional heat generation from frictional interaction within the fluid particles within the flow channel. Fig. 5 shows the effect of variations in couple stress inverse parameter, as shown in the plot, an increase in the couple stress inverse parameter is observed to enhance the fluid temperature distribution within the porous medium. This is physically true due to decreasing dynamic viscosity of the fluid. However, as the couple stress inverse parameter decreases, the dynamic viscosity of the fluid decreases and as such the degree of freedom of the fluid particles decreases this will ultimately decrease the fluid temperature within the channel.

In Fig. 6, the effect of couple stress inverse on the entropy generation rate is presented. As seen in the plot the entropy generation rate within the flow channel increases with an increase in the couple stress inverse parameter. This is expected since the couple stress inverse parameter increases both the velocity and temperature profiles as shown in Figs. 1 and 2. Thus the effect of couple stress on the thermo-fluid model is to maximize the exergy of the system since the viscosity of the fluid increases as the couple stress inverse parameter decreases. Moreover, as shown in Fig. 3, excessive increase of the concentration of the reacting fluid may increase the fluid temperature distribution significantly. As a result the rise in the Frank-Kameneskii parameter is observed to promote destruction of the useful work as shown in Fig. 7. Hence, this is one of the parameters to be monitored to discourage wastage of energy in the flow channel. Similar behavior is observed in Fig. 8 with an increase in the viscous heating parameter due to additional heat generated from the conversion of the kinetic energy of the fluid particle to heat energy.

Figs. 9–12 show the variations of parameters on the heat irreversibility ratio within the flow channel. In all the figures, irreversibility due to fluid friction dominates over irreversibility due to fluid friction at the centerline of the channel. However, it is observed in Fig. 9 that at the walls heat transfer irreversibility dominates over irreversibility due to fluid friction but as the couple stress inverse parameter increases, this dominance weakens at the walls. Also, Fig. 10 explains the influence of viscous heating parameter on the irreversibility ratio. As observed, an increase the viscous heating parameter increases, fluid friction between fluid particles increases, therefore, irreversibility due to fluid friction dominates over heat transfer irreversibility at the walls. Fig. 11 represents the effect of activation energy parameter on the irreversibility ratio, the graphical result shows that as the activation energy increases, irreversibility due to heat transfer dominates over fluid friction. Finally, Fig. 12 depicts the effect of Frank-Kameneskii parameter on the irreversibility ratio and the result shows that irreversibility due to heat transfer dominates over fluid friction irreversibility, this is true due to heat liberated

from the exothermic chemical reaction that is transferred to the isothermal wall.

5. Conclusion

In the present paper, the entropy generation rate in the steady flow of couple stress fluid through a porous space with nonlinear internal heat generation is considered. The equations governing the couple stress fluid flow has been proposed using the Brinkman model. The governing non-linear problem that comprised the balance laws of mass and momentum has been solved using regular perturbation method and used to compute the entropy generation rate and Bejan number. The significant contributions of the couple stress parameter on velocity, temperature, entropy generation rate and irreversibility ratio are pointed out.

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